Algebra

The Real Numbers

SECOND PREP

First Term 2019

LESSON (1) The cube root of a Rational number

The cube root of a rational number

Definition

The cube root of the rational number "a" is the number whose cube equal to a

The cube root of the rational number "a" is denoted by $\sqrt[3]{a}$

For example :

- $\sqrt[3]{8} = 2$ because $(2)^3 = 8$
- $\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$

Notice that : $\sqrt[3]{0} = 0$

i.e. • The cube root of any number has the same sign of this number.

Remarks

1. $\sqrt[3]{a^3} = a$ For example : $\sqrt[3]{5^3} = 5$, $\sqrt[3]{(-5)^3} = -5$
2. $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$ For example : $\sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2$

Remark

If "a" is a perfect cube number ,

then the equation : $x^3 = a$ has a unique solution in $\mathbb{Q}$, which is $\sqrt[3]{a}$

For example:
- The equation : $x^3 = 8$ has a unique solution in $\mathbb{Q}$ which is $\sqrt[3]{8} = 2$
- The equation : $x^3 = 9$ has no solution in $\mathbb{Q}$ because 9 is not a perfect cube.

Applications

Remember that :

- The volume of a cube = the edge length × itself × itself
- The area of one face of a cube = the edge length × itself
- The lateral area of a cube = the area of one face × 4
- The total area of a cube = the area of one face × 6

For example: If the volume of a cube is 8 cm^3 , then :

- The edge length = $\sqrt[3]{8} = 2$ cm.
- The area of one face = 2 × 2 = 4 cm^2
- The lateral area = 4 × 4 = 16 cm^2
- The total area = 4 × 6 = 24 cm^2

[A] Find the value of X in each of the following :

1) $\sqrt[3]{X} = 5$	2) $\sqrt[3]{X} = -\frac{1}{4}$	3) $\sqrt[3]{X} = -\sqrt{4}$	4) $\sqrt[3]{X} - 3 = 1$
$X = 5^3$ $X = 125$	$X = (-\frac{1}{4})^3$ $X = -\frac{1}{64}$	$\sqrt[3]{X} = -2$ $X = (-2)^3$ $X = -8$	$\sqrt[3]{X} = 3 + 1$ $\sqrt[3]{X} = 4$ $X = 4^3$ $X = 64$

[B] Find the S.S of each of the following equations in Q :

1) $X^3 = 64$	2) $X^3 + 5 = 32$	3) $\frac{1}{5}X^3 = -200$	4) $(3X + 1)^3 = -8$
$X = \sqrt[3]{64}$ $X = 4$ S.S = { 4 }	$X^3 = 32 - 5$ $X^3 = 27$ $X = \sqrt[3]{27}$ $X = 3$ S.S = { 3 }	$X^3 = -200 \div \frac{1}{5}$ $X = -1000$ $X = \sqrt[3]{1000}$ $X = 10$ S.S = { 10 }	$(3X + 1) = \sqrt[3]{-8}$ $3X+1 = -2$ $X = \frac{-2-1}{3}$ $X = -1$ S.S = { –1 }

5) $2X^3 - 5 = X^3 + 3$	6) $(X + 3)^3 = 343$	7) $(5X - 2)^3 + 10 = 18$
$2X^3 - X^3 = 5 + 3$ $X^3 = 8$ $X = \sqrt[3]{8}$ $X = 2$ S.S = { 2 }	$X + 3 = \sqrt[3]{343}$ $X + 3 = 7$ $X = 7 - 3$ $X = 4$ S.S = { 4 }	$(5X - 2)^3 = 18 - 10$ $(5X - 2)^3 = 8$ $5X - 2 = \sqrt[3]{8} = 2$ $X = \frac{2+2}{5} = \frac{4}{5}$ S.S = { $\frac{4}{5}$ }

EXERCISES

[A] Complete the Following :

1	📖 $\sqrt[3]{a^3} =$ …………
2	$\sqrt[3]{-8} =$ ……………
3	$\lvert \sqrt[3]{-125} \rvert =$ ………
4	📖 $\lvert \sqrt[3]{-125} \rvert = \sqrt{\text{………}}$
5	$\sqrt[3]{27} - \sqrt[3]{-27} =$ …………
6	$-\sqrt[3]{-1} - \sqrt{1} =$ …………
7	$\sqrt[3]{64 + \text{…………}} = 5$
8	$\sqrt[3]{\text{………}} = 4$
9	$\sqrt{16} = \sqrt[3]{\text{………}}$
10	$\sqrt[3]{64} = \sqrt{\text{………}}$
11	If : $\sqrt[3]{64} = \sqrt{x}$, then $2x =$ ……………
12	If : $x^2 = 5$, then $(x + \sqrt{5})^2 =$ ……… or ………
13	$\frac{x^3}{y^3} = \frac{1}{64}$, then $\left(\frac{y}{x}\right)^2 =$ ………
14	If $8 = \sqrt[3]{x}$, then $x =$ ………
15	If $\sqrt[3]{x} = -\sqrt{4}$, then $x =$ …………

16	If $x^2 - y^2 = 60$ and $x + y = 5$, then $x - y =$ ....
17	The solution set for the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is ....
18	The solution set of the equation : $x^2 + 4 = 0$ in $\mathbb{R}$ is ....
19	The solution set of the equation : $x^2 + 9 = 0$ in $\mathbb{Q}$ is ....
20	The S.S. of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is ....
21	The solution set of the equation : $(x^2 + 3)(x^2 + 1) = 0$ where $x \in \mathbb{R}$ is ....
22	The S.S. of the equation : $(x^2 - 1)(x + 5) = 0$ in $\mathbb{R}$ is ....
23	The S.S. of the equation : $(x^2 + 1)(x - 5) = 0$ in $\mathbb{R}$ is ....
24	The S.S. of the equation : $x^3 + 1 = 2$ in $\mathbb{R}$ is ....
25	The S.S. of the equation : $x(x^3 - 1) = 0$ in $\mathbb{R}$ is ....
26	The S.S. of the equation : $(x^2 + 3)(x^3 + 1) = 0$ is ....

[B] Essay Problems :

1	**Find the value of x in each of the following :** 🕮 $x^3 = -8$ **Exercise (1) Question (5) (5)**
2	**Find the S.S. of each of the following equations in $\mathbb{Q}$:** 🕮 $x^3 + 27 = 0$ **Exercise (1) Question (6) (1)**
3	**Find the S.S. of each of the following equations in $\mathbb{Q}$:** 🕮 $8x^3 + 7 = 8$ **Exercise (1) Question (6) (2)**
4	Find the edge length of a cube with volume = $15\frac{5}{8}$ cm^3. «2.5 cm.» **Exercise (1) Question (8)**

HOMEWORK

[A] Choose the correct answer:

1	$\left(2\sqrt[3]{2}\right)^3 = \dots\dots$ (a) 4 (b) 8 (c) 16 (d) 40
2	📖 $\sqrt[3]{(-8)^2} = \dots\dots$ (a) 2 (b) -2 (c) 4 (d) -4
3	$\sqrt{8} - \sqrt{2} = \dots\dots$ (a) $\sqrt{6}$ (b) 2 (c) $\sqrt{2}$ (d) 1
4	$\sqrt{25} - \sqrt[3]{-125} = \dots\dots$ (a) 10 (b) zero (c) 5 (d) ± 5
5	$-2\sqrt{3} \times \sqrt{3} = \dots\dots$ (a) $-2\sqrt{3}$ (b) -6 (c) $2\sqrt{3}$ (d) 6
6	$\sqrt{3}\left(\sqrt{11} + \sqrt{3}\right) = \dots\dots$ (a) $3\sqrt{11} + 2$ (b) $\sqrt{33} + 3$ (c) $11\sqrt{3} + 2$ (d) $2\sqrt{11} + 3$
7	$\sqrt{9} + \sqrt[3]{-27} = \dots\dots$ (a) 0 (b) -6 (c) -9 (d) ± 6
8	$\sqrt[3]{-8} + \sqrt{4} = \dots\dots$ (a) 4 (b) -4 (c) zero (d) 8
9	$\sqrt{25} = \sqrt[3]{\dots\dots}$ (a) 5 (b) 15 (c) 125 (d) -5

10	If : $\sqrt[3]{y} = -\sqrt{9}$, then y = ... (a) 3 (b) – 3 (c) – 27 (d) 27
11	$\sqrt{25} + \sqrt[3]{-27} = \sqrt{\dots\dots\dots}$ (a) 8 (b) 4 (c) 2 (d) 5
12	$\sqrt[3]{27} = \sqrt{x+3}$, then x = ... (a) 3 (b) 6 (c) 9 (d) 12
13	If $x^3 = 64$, then $\sqrt{x}$ = ... (a) 4 (b) – 4 (c) 2 (d) – 2
14	The solution set for the equation : $x^2 = 2$ in ℝ is .... (a) $\{\sqrt{2}\}$ (b) $\{-\sqrt{2}\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\{2\}$
15	The S.S. of the equation : $x^2 + 3 = 0$ in ℝ is .... (a) $\varnothing$ (b) $-\sqrt{3}$ (c) $\sqrt{3}$ (d) $\pm\sqrt{3}$
16	The S.S. of the equation : $x^2 + 5 = 9$ where $x \in$ ℚ is .... (a) {4} (b) {– 2 , 2} (c) $\varnothing$ (d) {13}
17	The S.S. of the equation : $x^3 + 8 = 0$ in ℝ is .... (a) {2} (b) $\{2\sqrt{2}\}$ (c) {– 2} (d) {2 , – 2}
18	The solution set for the equation : $x^3 + 9 = 8$ in ℝ is .... (a) {8} (b) {9} (c) {3} (d) {– 1}
19	The S.S. of the equation : $x^3 + 27 = 0$ in ℝ is .... (a) {3} (b) {– 3} (c) $\{3\sqrt{3}\}$ (d) $\{\pm 3\sqrt{3}\}$
20	The S.S. in ℝ of the equation : $x^3 + 11 = 12$ in ℝ is .... (a) {11} (b) {12} (c) {1} (d) {3}

[B] Essay Problems :

1	Find the value of x in each of the following : 🕮 $\sqrt[3]{x} = 5$ **Exercise (1) Question (5) (1)**
2	Find the value of x in each of the following : 🕮 $x^3 = -8$ **Exercise (1) Question (5) (5)**
3	Find the S.S. of each of the following equations in $\mathbb{Q}$: 🕮 $x^3 + 27 = 0$ **Exercise (1) Question (6) (1)**
4	Find the S.S. of each of the following equations in $\mathbb{Q}$: 🕮 $8x^3 + 7 = 8$ **Exercise (1) Question (6) (2)**
5	Find the S.S. of each of the following equations in $\mathbb{Q}$: 🕮 $(x + 3)^3 = 343$ **Exercise (1) Question (6) (5)**
6	Find the S.S. of each of the following equations in $\mathbb{Q}$: 🕮 $(5x - 2)^3 + 10 = 18$ **Exercise (1) Question (6) (8)**
7	Find the edge length of a cube with volume = $15\frac{5}{8}$ cm^3. «2.5 cm.» **Exercise (1) Question (8)**
8	🕮 Find the inner edge length of a cube vessel with capacity of one litre. «10 cm.» **Exercise (1) Question (12)**
9	🕮 Find the diameter length of a sphere whose volume = $\frac{1372}{81}\pi$ cube unit. «$\frac{14}{3}$ length unit » **Exercise (1) Question (13)**

LESSON (2) The Set Of Irrational Numbers ($\grave{Q}$)

1 The square root of the perfect square rational number is a rational number.

For example :

$\sqrt{1}$, $\sqrt{\frac{1}{9}}$, $\sqrt{\frac{25}{4}}$, $\sqrt{0.09}$, ... that are rational numbers.

- But the square root of the rational number which is not a perfect square is not a rational number.

For example :

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{\frac{2}{5}}$ ... they are not rational numbers

Because there is no rational number whose square is : 2 or 3 or $\frac{2}{5}$ or ...

2 The cube root of the perfect cube rational number is a rational number.

For example :

$\sqrt[3]{8}$, $\sqrt[3]{-64}$, $\sqrt[3]{0.027}$, .. they are rational numbers.

- But the cube root of the rational number which is not a perfect cube is not a rational number.

For example :

$\sqrt[3]{2}$, $\sqrt[3]{4}$, $\sqrt[3]{\frac{5}{8}}$, ... they are not rational numbers.

Because there is no rational number whose cube equals 2 or 4 or $\frac{5}{8}$ or ....

3 The numbers $\frac{22}{7}$, 3.14 , 3.142 , ... are rational numbers , each of them represents an approximating value of the number π

But the number π which denotes the ratio between the circumference of the circle and its diameter length is not a rational number.

From the previous, we deduce that :

There is another set of numbers which are not rational numbers. This set is called "the set of irrational numbers" and it is denoted by $\grave{\mathbb{Q}}$

Notice that :

$\mathbb{Q}$ and $\grave{\mathbb{Q}}$ are disjoint sets. **i.e.** $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

i.e. The irrational number is represented by an infinite decimal and not recurring.

Remark

- $(\sqrt{a})^2 = \sqrt{a} \times \sqrt{a} = a$, where $a \geq 0$ **For example :** $(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = 2$
- $(\sqrt[3]{a})^3 = \sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$, where $a \in \mathbb{Q}$ **For example :** $(\sqrt[3]{-7})^3 = \sqrt[3]{-7} \times \sqrt[3]{-7} \times \sqrt[3]{-7} = -7$

Remark

From the previous , we can deduce that :

Each irrational number lies between two rational numbers.

Example (3) **Prove that :**

1 $\sqrt{3}$ lies between 1.7 and 1.8 **2** $\sqrt[3]{12}$ lies between 2.2 and 2.3

Solution

1 $\because (\sqrt{3})^2 = \sqrt{3} \times \sqrt{3} = 3$, $(1.7)^2 = 2.89$, $(1.8)^2 = 3.24$

$\because 2.89 < 3 < 3.24$ $\therefore \sqrt{2.89} < \sqrt{3} < \sqrt{3.24}$ $\therefore 1.7 < \sqrt{3} < 1.8$

i.e. $\sqrt{3}$ lies between 1.7 and 1.8

You can solve the problem using the calculator as follows :

$\because \sqrt{3} \simeq 1.73$ $\because 1.7 < 1.73 < 1.8$

$\therefore 1.7 < \sqrt{3} < 1.8$ $\therefore \sqrt{3}$ lies between 1.7 and 1.8

2 $\because (\sqrt[3]{12})^3 = \sqrt[3]{12} \times \sqrt[3]{12} \times \sqrt[3]{12} = 12$, $(2.2)^3 = 10.648$, $(2.3)^3 = 12.167$

$\because 10.648 < 12 < 12.167$ $\therefore \sqrt[3]{10.648} < \sqrt[3]{12} < \sqrt[3]{12.167}$

$\therefore 2.2 < \sqrt[3]{12} < 2.3$

i.e. $\sqrt[3]{12}$ lies between 2.2 and 2.3

You can solve the problem using the calculator as follows :

$\sqrt[3]{12} \simeq 2.289$ $\because 2.2 < 2.289 < 2.3$

$\therefore 2.2 < \sqrt[3]{12} < 2.3$ $\therefore \sqrt[3]{12}$ lies between 2.2 and 2.3

Representing an irrational number on the number line

Therefore we can deduce that :

Each irrational number can be represented by a point on the number line.

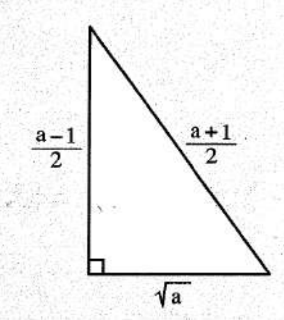
Generally

To draw a line segment with length $\sqrt{a}$ length unit where a > 1 ,

draw a right-angled triangle in which

the length of one side of the right-angle = $\frac{a-1}{2}$ length unit

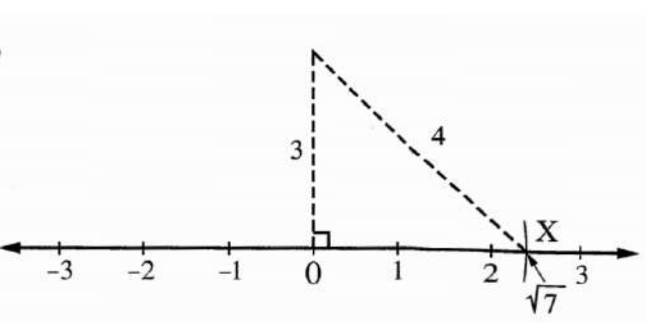
and the length of the hypotenuse = $\frac{a+1}{2}$ length unit.



Example (4)

Draw a line segment with length = $\sqrt{7}$ length unit , then use it to determine the points which represent the following numbers on the number line :

Using the compasses with a distance equal to the length of $\overline{BC}$ taking O as a centre , draw an arc to cut the number line on the right side of O at the point X , then X is the point which represents $\sqrt{7}$

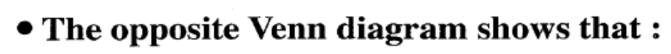


LESSON (3) The Set Of Real Numbers R

The set of real numbers

It is the set obtained from the union of the set of rational numbers and the set of irrational numbers. It is denoted by $\mathbb{R}$

i.e. $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$ (as shown in the opposite figure)

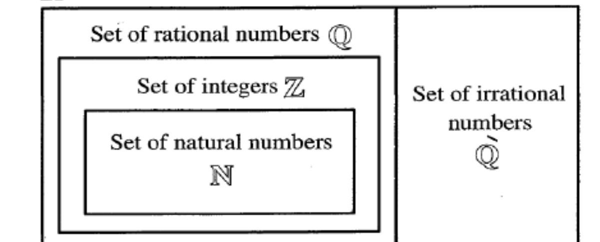


Noticing that : $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

- **The opposite Venn diagram shows that :**

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

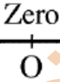
and $\grave{\mathbb{Q}} \subset \mathbb{R}$





$\mathbb{R}_- = \{x : x \in \mathbb{R} , x < \text{zero}\}$

$\mathbb{R}_+ = \{x : x \in \mathbb{R} , x > \text{zero}\}$

Remarks

- $\mathbb{R}_+ \cap \mathbb{R}_- = \varnothing$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_-$
- The number zero is neither positive nor negative.
- $\mathbb{R}_+ \cup \{0\} = \{x : x \in \mathbb{R}, x \geq 0\}$ and it is called the set of the non-negative real numbers.
- $\mathbb{R}_- \cup \{0\} = \{x : x \in \mathbb{R}, x \leq 0\}$ and it is called the set of the non-positive real numbers.
- The set of real numbers without zero (The non-zero real numbers) is denoted by $\mathbb{R}^*$

  i.e. $\mathbb{R}^* = \mathbb{R} - \{0\} = \mathbb{R}_+ \cup \mathbb{R}_-$

EXERCISES

[A] Complete the Following :

1	If : $x < -\sqrt{7} < x + 1$, then $x =$ .... (where x is an integer)
2	If : $x < \sqrt{15} < x + 1$, $x \in \mathbb{Z}$, then $x =$ ....
3	If : $x < \sqrt{19} < x + 1$, then $x =$ ....
4	If $x < \sqrt{20} < x + 1$, $x \in \mathbb{Z}$, then $x =$ ....
5	If $x < \sqrt{10} < x + 1$, $x \in \mathbb{Z}_+$, then $x =$ ....
6	$\mathbb{Q} \cap \grave{\mathbb{Q}} =$ ....
7	$\mathbb{Q} \cup \grave{\mathbb{Q}} =$ ....
8	$\mathbb{R}_+ \cup \mathbb{R}_- =$ ....
9	The multiplicative inverse of $\frac{\sqrt{5}}{10}$ is ....
10	The multiplicative inverse of the number : $(\sqrt{3} + \sqrt{2})$ is ....

11	The additive inverse of : $3 - 2\sqrt{2}$ is ……………
12	$\sqrt[3]{64 + \dots\dots\dots} = 5$
13	$\sqrt[3]{\dots\dots\dots} = 4$
14	$\sqrt{16} = \sqrt[3]{\dots\dots\dots}$
15	$\sqrt[3]{64} = \sqrt{\dots\dots\dots}$
16	If : $\sqrt[3]{64} = \sqrt{x}$, then $2x =$ ……………
18	The solution set of the equation : $x^2 + 9 = 0$ in $\mathbb{Q}$ is ……………
19	The S.S. of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is ……………

[B] Essay Problems :

1	**Find an approximated value for each of the following numbers :** $\sqrt[3]{-9}$ "to the nearest tenth". **Exercise (2) Question (8) (1)**
2	**If x is an integer , find the value of x in each of the following cases :** $x < \sqrt{2} < x + 1$ **Exercise (2) Question (4) (1)**
3	**If x is an integer , find the value of x in each of the following cases :** $x < \sqrt{80} < x + 1$ «8» **Exercise (2) Question (4) (2)**
4	**Determine the point that represents each of the following numbers on the number line :** (1) $\sqrt{3}$ (2) $-\sqrt{11}$ (3) $\sqrt{10}$ **Exercise (2) Question (9) (4)**

HOMEWORK

[A] Choose the correct answer:

1	If : $\frac{3}{a+2}$ is a rational number then a ≠ .... (a) 3 (b) 5 (c) – 2 (d) zero
2	If $n \in \mathbb{Z}_+$, $n < \sqrt{26} < n + 1$, then n = ... (a) 25 (b) 5 (c) –5 (d) 24
3	The irrational number in the following numbers is .... (a) $\sqrt{\frac{1}{9}}$ (b) $\sqrt{\frac{1}{4}}$ (c) $\sqrt{3}$ (d) $\sqrt[3]{27}$
4	The irrational number in the following numbers is .... (a) $\sqrt{\frac{1}{4}}$ (b) $\sqrt[3]{8}$ (c) $\sqrt{\frac{4}{9}}$ (d) $\sqrt{2}$
5	The irrational number lies between 2 and 3 is .... (a) $\sqrt{10}$ (b) $\sqrt{7}$ (c) 2.5 (d) $\sqrt{3}$
6	The irrational number lies between 3 and 4 is .... (a) 3.5 (b) $\frac{1}{8}$ (c) $\sqrt{20}$ (d) $\sqrt{13}$
7	🕮 The area of a square whose side length is $\sqrt{3}$ cm. = ... cm^2 (a) $4\sqrt{3}$ (b) 9 (c) 3 (d) 6
8	🕮 The square whose area is 10 cm^2, its side length is .... cm. (a) 5 (b) – 5 (c) $\sqrt{10}$ (d) $-\sqrt{10}$
9	The multiplicative inverse of $\frac{\sqrt{3}}{3}$ is .... (a) $\sqrt{3}$ (b) 1 (c) 3 (d) $-\sqrt{3}$

10 The multiplicative inverse of $\frac{\sqrt{5}}{10}$ is ....

(a) $-\sqrt{10}$ (b) $\sqrt{5}$ (c) $-2\sqrt{5}$ (d) $2\sqrt{5}$

11 The multiplicative inverse of the number $\sqrt{5}$ is ....

(a) $-\sqrt{5}$ (b) $\frac{\sqrt{5}}{5}$ (c) $5\sqrt{5}$ (d) $\frac{5}{\sqrt{5}}$

12 $\mathbb{Q} \cap \grave{\mathbb{Q}} =$ ....

(a) $\{0\}$ (b) $\varnothing$ (c) $\mathbb{R}$ (d) $\mathbb{Q}$

13 If : $\sqrt[3]{y} = -\sqrt{9}$, then y = ...

(a) 3 (b) – 3 (c) – 27 (d) 27

14 $\sqrt{25} + \sqrt[3]{-27} = \sqrt{\dots\dots\dots\dots}$

(a) 8 (b) 4 (c) 2 (d) 5

15 $\sqrt[3]{27} = \sqrt{x+3}$, then $x =$ ....

(a) 3 (b) 6 (c) 9 (d) 12

16 The solution set for the equation : $x^2 = 2$ in $\mathbb{R}$ is ....

(a) $\{\sqrt{2}\}$ (b) $\{-\sqrt{2}\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\{2\}$

17 $\sqrt[3]{27} = \sqrt{x+3}$, then $x =$ ....

(a) 3 (b) 6 (c) 9 (d) 12

18 If $x^3 = 64$, then $\sqrt{x} =$ ....

(a) 4 (b) – 4 (c) 2 (d) – 2

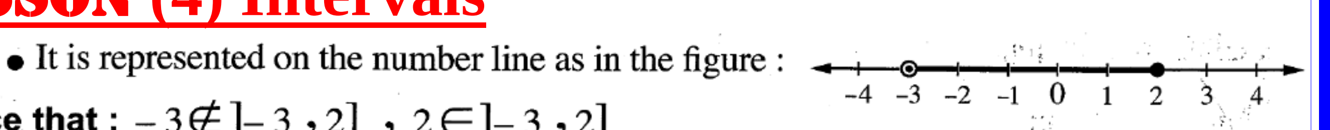
19 The solution set for the equation : $x^2 = 2$ in $\mathbb{R}$ is ....

(a) $\{\sqrt{2}\}$ (b) $\{-\sqrt{2}\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\{2\}$

[B] Essay Problems :

1	Find an approximated value for each of the following numbers : $\sqrt{11}$ "to the nearest hundredth". **Exercise (2) Question (8) (1)**
2	Find an approximated value for each of the following numbers : $\sqrt[3]{-9}$ "to the nearest tenth". **Exercise (2) Question (8) (1)**
3	If x is an integer , find the value of x in each of the following cases : $x < \sqrt{2} < x + 1$ **Exercise (2) Question (4) (1)**
4	If x is an integer , find the value of x in each of the following cases : $x < \sqrt{80} < x + 1$ «8» **Exercise (2) Question (4) (2)**
5	Prove that : $\sqrt{2}$ is included between 1.4 and 1.5 **Exercise (2) Question (9) (1)**
6	Prove that : $\sqrt[3]{15}$ is included between 2.4 and 2.5 **Exercise (2) Question (9) (4)**
7	Determine the point that represents each of the following numbers on the number line : (1) $\sqrt{3}$ (2) $-\sqrt{11}$ (3) $\sqrt{10}$ **Exercise (2) Question (9) (4)**

LESSON (4) Intervals



- It is represented on the number line as in the figure :

Notice that : $-3 \notin]-3 , 2]$, $2 \in]-3 , 2]$

Types of intervals		The interval	Expression by distinguished property	Representation on the number line	Notice that
The limited intervals	Closed	$[a , b]$	$\{x : x \in \mathbb{R} , a \le x \le b\}$	a ● ——— ● b	• $a \in [a , b]$ • $b \in [a , b]$
	Opened	$]a , b[$	$\{x : x \in \mathbb{R} , a < x < b\}$	a ○ ——— ○ b	• $a \notin]a , b[$ • $b \notin]a , b[$
	half opened (half closed)	$[a , b[$	$\{x : x \in \mathbb{R} , a \le x < b\}$	a ● ——— ○ b	• $a \in [a , b[$ • $b \notin [a , b[$
		$]a , b]$	$\{x : x \in \mathbb{R} , a < x \le b\}$	a ○ ——— ● b	• $a \notin]a , b]$ • $b \in]a , b]$
The unlimited intervals		$[a , \infty[$	$\{x : x \in \mathbb{R} , x \ge a\}$	a ● ——→	$a \in [a , \infty[$
		$]a , \infty[$	$\{x : x \in \mathbb{R} , x > a\}$	a ○ ——→	$a \notin]a , \infty[$
		$]-\infty , a]$	$\{x : x \in \mathbb{R} , x \le a\}$	←—— ● a	$a \in]-\infty , a]$
		$]-\infty , a[$	$\{x : x \in \mathbb{R} , x < a\}$	←—— ○ a	$a \notin]-\infty , a[$

Remarks

1. $\mathbb{R} =]-\infty , \infty[$
2. $\mathbb{R}_+ =]0 , \infty[$
3. $\mathbb{R}_- =]-\infty , 0[$
4. The set of non-negative real numbers $= \mathbb{R}_+ \cup \{0\} = [0 , \infty[$
5. The set of non-positive real numbers $= \mathbb{R}_- \cup \{0\} =]-\infty , 0]$

1	$X \cup Y$	Smaller , Bigger	] - ∞ , ∞ [= R
2	$X \cap Y$	Bigger, Smaller	[- 4 , 3]
3	$X - Y$	Smaller , Small	] - ∞ , - 4 [
4	$Y - X$	Big , Bigger	] 3 , ∞ [
5	$\grave{X}$	Replace	] 3 , ∞ [
6	$\grave{Y}$	Replace	] - ∞ , - 4 [

EXERCISES

[A] Complete the Following :

1	$[1, 3] \cup [2, 5[=$ ....
2	$]1, 3] \cup [2, 5] =$ ....
3	$]-\infty, 1] \cup [-4, \infty[=$ ....
4	$]3, 5[\cup \{3, 5\} =$ ....
5	$]-2, 2] \cup \{-2, 0\} =$ ....
6	$]5, 7[\cup \{5, 7\} =$ ....
7	$\mathbb{N} \cap]1, 2[=$ ....
8	$]1, 7[\cap]3, 5[=$ ....
9	$[-3, 1[\cap [-1, 4[=$ ....
10	$[-2, 5] \cap]4, 6] =$ ....
11	$]-3, 5] \cap [0, 3[=$ ....
12	$[1, 5] - \{1, 5\} =$ ....
13	$[2, 5] - \{5\} =$ ....
14	$[2, 5] - \{2, 5\} =$ ....

15	$[3, 4] - \{3, 4\} =$ ....
16	$[3, 5] - \{3\} =$ ....
17	$[3, 7[-]-2, 5] =$ ....
18	$[-4, 6] -]-4, 6[=$ ....
19	$[-1, 5] -]-1, 5[=$ ....
20	$[2, 7] -]2, 7[=$ ....

[B] Essay Problems :

1	If $X = [-1, 4]$, $Y = [3, \infty[$, $Z = \{3, 4\}$, find using the number line : (1) $X \cup Y$ (2) $X \cap Y$ (3) $X - Z$ **2016 Exam (1) Question (4) (a)**
2	If $X = [3, \infty[$, $Y =]-4, 8[$ Find : (1) $X \cup Y$ (2) $X \cap Y$ (3) $\grave{X}$ **2016 Exam (4) Question (3) (a)**
3	Find each of the following : (1) $[0, 5] \cup [3, 8[$ (2) $[1, 5] \cap]-2, 3]$ **2016 Exam (8) Question (5) (a)**
4	If $X = [-2, 3]$, $Y = [1, 5[$, then find by using the number line : $X \cup Y$, $X - Y$ **2016 Exam (12) Question (3) (b)**

HOMEWORK

[A] Choose the correct answer:

1	$\sqrt[3]{8}$ .... $]-\infty , 4[$ (a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$
2	$5 \in$ .... (a) $]5 , \infty[$ (b) $]-\infty , 5[$ (c) $(3 , 5)$ (d) $[-5 , \infty[$
3	The opposite figure represents the interval .... (a) $[-4 , 8[$ (b) $[8 , -4]$ (c) $[-4 , 8]$ (d) $]-4 , 8[$
4	$\mathbb{R} =$ .... (a) $\mathbb{R}_+ \cap \mathbb{R}_-$ (b) $\mathbb{R}_+ \cup \mathbb{R}_-$ (c) $]-\infty , \infty[$ (d) $\mathbb{Q} \cap \mathbb{Q}'$
5	$\mathbb{R}_+ =$ .... (a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$
6	$\mathbb{R}_- =$ .... (a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$
7	The set of non-negative real numbers = ... (a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$
8	The set of non-positive real numbers = ... (a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$
9	$]-1 , 3] \cap [-3 , -1] =$ .... (a) $\varnothing$ (b) $\{-3\}$ (c) $\{-1\}$ (d) $\{3\}$

10	$[1,5] \cap]-2,3] = \cdots\cdots\cdots$ (a) $\{1,3\}$ (b) $]1,3[$ (c) $[1,3]$ (d) $[1,3[$
11	$]-3,5[\cap [0,3[= \cdots\cdots\cdots$ (a) $[0,3]$ (b) $[0,3[$ (c) $]-3,0[$ (d) $[3,5[$
12	$[2,7] - \{2,7\} = \cdots\cdots\cdots$ (a) $[1,6]$ (b) $\varnothing$ (c) $]2,7[$ (d) $\{0\}$
13	$[-2,5] - \{-2,6\} = \cdots\cdots\cdots$ (a) $]-2,5[$ (b) $]-2,6[$ (c) $]-2,5]$ (d) $[-2,5[$
14	$[-3,7] - \{-3,7\} = \cdots\cdots\cdots$ (a) $[-3,7[$ (b) $]-3,7]$ (c) $]-3,7[$ (d) $(0,0)$

[B] Essay Problems :

1	If $X = [-1,4]$, $Y = [3,\infty[$, $Z = \{3,4\}$, **find using the number line :** (1) $X \cup Y$ (2) $X \cap Y$ (3) $X - Z$ **2016 Exam (1) Question (4) (a)**
2	If $X = [-2,1]$ and $Y = [0,\infty[$ **Find :** (1) $X \cap Y$ (2) $X \cup Y$ **2016 Exam (3) Question (4) (a)**
3	If $X = [3,\infty[$, $Y =]-4,8[$ **Find :** (1) $X \cup Y$ (2) $X \cap Y$ (3) $\grave{X}$ **2016 Exam (4) Question (3) (a)**

4	If $X = [-1, 4]$ and $Y = [2, 7]$, then find each of : (1) $X \cap Y$ (2) $Y \cup X$ **2016 Exam (5) Question (4) (b)**
5	If $X = [-2, 1]$, $Y = [0, \infty[$ Find : (1) $X \cap Y$ (2) $X \cup Y$ (3) $Y - X$ **2016 Exam (6) Question (4) (a)**
6	If $X = [-1, 4]$, $Y = [3, \infty[$, find using the number line each of : (1) $X \cup Y$ (2) $X - Y$ **2016 Exam (7) Question (3) (b)**
7	Find each of the following : (1) $[0, 5] \cup [3, 8[$ (2) $[1, 5] \cap]-2, 3]$ **2016 Exam (8) Question (5) (a)**
8	If $X = [-2, 3]$, $Y = [1, 5[$, then find by using the number line : $X \cup Y$, $X - Y$ **2016 Exam (12) Question (3) (b)**
9	If $X = [-2, 4]$ and $Y =]2, \infty[$, find each of the following using the number line : (1) $X \cap Y$ (2) $X - Y$ **2016 Exam (13) Question (3) (a)**
10	If $X = [-1, 4]$, $Y = [2, 7]$, then find each of the following by using the number line : (1) $X \cup Y$ (2) $X \cap Y$ (3) $X - Y$ **2016 Exam (14) Question (4) (b)**
11	If $X = [1, 5]$, $Y = [2, 7]$ Find by using the number line : (1) $X \cap Y$ (2) $X \cup Y$ **2016 Exam (15) Question (4) (b)**

LESSON (5) Operations on the Real Numbers

Properties of addition of real numbers

The additive neutral :

For every $a \in \mathbb{R}$ it will be $a + 0 = 0 + a = a$

i.e. Zero is the additive neutral.

For example : $\sqrt{2} + 0 = 0 + \sqrt{2} = \sqrt{2}$, $-\sqrt[3]{5} + 0 = 0 + (-\sqrt[3]{5}) = -\sqrt[3]{5}$

The additive inverse of every real number :

For every $a \in \mathbb{R}$ there is $(-a) \in \mathbb{R}$ where $a + (-a) =$ zero (the additive neutral)

For example :

- The additive inverse of the number $\sqrt{3}$ is $-\sqrt{3}$ and vice versa because $\sqrt{3} + (-\sqrt{3}) = 0$
- The additive inverse of the number $2 + \sqrt{5}$ is $-(2 + \sqrt{5})$ and equals $-2 - \sqrt{5}$
- The additive inverse of the number $3 - \sqrt{2}$ is $-(3 - \sqrt{2})$ and equals $\sqrt{2} - 3$
- The additive inverse of the number zero is itself.

The Properties of multiplication operation of real numbers

The multiplicative neutral :

For every $a \in \mathbb{R}$ it will be $a \times 1 = 1 \times a = a$

i.e. One is the multiplicative neutral in $\mathbb{R}$

For example :

- $\sqrt[3]{5} \times 1 = 1 \times \sqrt[3]{5} = \sqrt[3]{5}$

The multiplicative inverse of any non-zero real number :

For every real number $a \neq 0$, there is a real number $\frac{1}{a}$ where $a \times \frac{1}{a} = 1$ which is the multiplicative neutral.

For example :

- The multiplicative inverse of $\sqrt{3}$ is $\frac{1}{\sqrt{3}}$ because $\sqrt{3} \times \frac{1}{\sqrt{3}} = 1$
- The multiplicative inverse of $-\frac{\sqrt{2}}{5}$ is $-\frac{5}{\sqrt{2}}$

Notice that :

Both the number and its multiplicative inverse have the same sign.

- The multiplicative inverse of the number 1 is itself and also the multiplicative inverse of -1 is itself.

Notice that :

There is no multiplicative inverse for the number zero because $\frac{1}{\text{zero}}$ is meaningless (**i.e.** undefined)

Remark

- Since each non-zero real number has a multiplicative inverse then the division operation by any real number does not equal zero is possible in $\mathbb{R}$ and it is defined as

  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}^*$ it will be $a \div b = a \times \frac{1}{b}$

  i.e. The division operation $(a \div b)$ means multiplying the number a by the multiplicative inverse of the number b such that $b \neq 0$

Then we can deduce that :

The division operation in $\mathbb{R}$ is not commutative and it is not associative.

LESSON (6) Operations on the square roots

Remarks

1 $\sqrt{a^2+b^2} \neq a+b$, $\sqrt{a^2-b^2} \neq a-b$

For example :

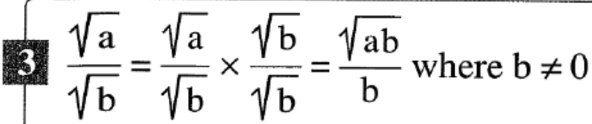
- $\sqrt{6^2+8^2} \neq 6+8$ because

  $\sqrt{6^2+8^2} = \sqrt{36+64} = \sqrt{100} = 10$
- $\sqrt{25-9} \neq 5-3$ because $\sqrt{25-9} = \sqrt{16} = 4$

2 $a\sqrt{b} = \sqrt{a^2 b}$

For example :

- $2\sqrt{\frac{1}{2}} = \sqrt{4 \times \frac{1}{2}} = \sqrt{2}$
- $15\sqrt{\frac{1}{3}} = 5 \times 3\sqrt{\frac{1}{3}} = 5\sqrt{9 \times \frac{1}{3}} = 5\sqrt{3}$

3 

$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$ where $b \neq 0$

This operation is carried out to make the denominator an integer.

For example :

- $\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$
- $\sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

Remarks

1 $\sqrt{a^2 + b^2} \neq a + b$, $\sqrt{a^2 - b^2} \neq a - b$

For example :

- $\sqrt{6^2 + 8^2} \neq 6 + 8$ because

  $\sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$
- $\sqrt{25 - 9} \neq 5 - 3$ because $\sqrt{25 - 9} = \sqrt{16} = 4$

2 $a\sqrt{b} = \sqrt{a^2 b}$

For example :

- $2\sqrt{\frac{1}{2}} = \sqrt{4 \times \frac{1}{2}} = \sqrt{2}$
- $15\sqrt{\frac{1}{3}} = 5 \times 3\sqrt{\frac{1}{3}} = 5\sqrt{9 \times \frac{1}{3}} = 5\sqrt{3}$

EXERCISES

[A] Complete the Following :

1	The multiplicative neutral in $\mathbb{R}$ is ………. and the additive neutral in $\mathbb{R}$ is ……….
2	The additive inverse of the number $1 - \sqrt{2}$ is ……….
3	The additive inverse of the number $\sqrt{5} - \sqrt{2}$ is ……………
4	The additive inverse of : $(\sqrt{7} - \sqrt{3})$ is ……….
5	The additive inverse of $\sqrt{48} - 5\sqrt{3}$ is ……….
6	The multiplicative inverse of the number $\frac{2\sqrt{3}}{5}$ is $\frac{\text{……….}}{6}$

7	The multiplicative inverse of $\frac{3}{\sqrt{3}}$ is ....
8	The multiplicative inverse of $\frac{2}{\sqrt{2}}$ is ....
9	The multiplicative inverse of the number $\frac{3}{\sqrt{3}}$ is $\frac{\dots\dots}{\sqrt{3}}$
10	The multiplicative inverse of $\frac{5}{\sqrt{5}}$ is $\frac{\dots\dots\dots}{\sqrt{5}}$
11	📖 If $a \in \mathbb{R}$ and $b \in \mathbb{R}$, then $a - b$ means the sum of the number a and .... of the number b
12	📖 If $a \in \mathbb{N}$, $b \in \mathbb{Q}$ and $c \in \mathbb{R}$, then $a + b + c \in$ ....
13	If : $a - b = 2\sqrt{5}$, the value of : $a(a-b)^3 + b(b-a)^3 =$ ....
14	$\frac{3\sqrt{2}}{2\sqrt{18}} =$ ....
15	$\sqrt{3} \times \sqrt{6} = 3 \times$ ....
16	$\frac{1}{2}\sqrt{48} = 2 \times$ ....
17	If $2\sqrt{27} - 2\sqrt{48} = x\sqrt{3}$, then $x =$ ....
18	📖 $\sqrt{5}$, $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, ... in the same pattern.
19	If $x^2 = \frac{8}{9}$, then x in the simplest form = ...
20	📖 If $x^2 = 5$, then $(x + \sqrt{5})^2 =$ .... or ....
21	If $\sqrt{x} = \sqrt{2} + 1$, then $x =$ ....

[B] Essay Problems :

1	Reduce to the simplest form : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ **2016 Exam (7) Question (3) (a)**
2	Find the value of : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ **2016 Exam (5) Question (4) (a)**
3	Find the simplest form of : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ **2016 Exam (9) Question (3) (a)**
4	Find in simplest form the expression : $\sqrt{50}+2\sqrt{18}-\sqrt{32}$ **2016 Exam (11) Question (3) (a)**
5	Find in the simplest form : $\sqrt{50}+\sqrt{18}-\sqrt{2}$ **2016 Exam (12) Question (3) (a)**
6	Simplify to the simplest form : $\sqrt{32}+\sqrt{72}-6\sqrt{\frac{1}{2}}$ **2016 Exam (14) Question (3) (a)**
7	Simplify to the simplest form : $\sqrt{75}+\sqrt{48}-3\sqrt{3}$ **2016 Exam (15) Question (3) (a)**

HOMEWORK

[A] Choose the correct answer:

1	The multiplicative inverse of the number $\sqrt{5}$ is ……… (a) $\frac{5}{\sqrt{5}}$ (b) $-\sqrt{5}$ (c) $\frac{\sqrt{5}}{5}$ (d) $5\sqrt{5}$
2	The multiplicative inverse of the number $\sqrt{7}$ is ……… (a) $-\sqrt{7}$ (b) $\frac{-1}{\sqrt{7}}$ (c) $\frac{\sqrt{7}}{7}$ (d) $\frac{7}{\sqrt{7}}$
3	The multiplicative inverse of the number $\sqrt{3}$ is ……… (a) 3 (b) $\frac{1}{3}$ (c) $-\sqrt{3}$ (d) $\frac{\sqrt{3}}{3}$
4	The multiplicative inverse of $-\sqrt{2}$ is …………… (a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{-\sqrt{2}}{2}$
5	The multiplicative inverse of the number $\frac{\sqrt{2}}{10}$ is …………… (a) $2\sqrt{5}$ (b) $-\frac{\sqrt{9}}{10}$ (c) $\frac{\sqrt{10}}{2}$ (d) $5\sqrt{2}$
6	The multiplication inverse of $\frac{\sqrt{3}}{3}$ is …………… (a) $\sqrt{3}$ (b) 1 (c) 3 (d) $-\sqrt{3}$
7	The multiplicative inverse of $\frac{\sqrt{3}}{6}$ is ……… (a) $\frac{-\sqrt{3}}{6}$ (b) $6\sqrt{3}$ (c) $2\sqrt{3}$ (d) $-2\sqrt{3}$
8	The multiplicative inverse of the number $\sqrt{5}$ is ……… (a) -5 (b) $\frac{-1}{5}$ (c) $\frac{5}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{5}$

9	The multiplicative inverse of the number $\frac{\sqrt{2}}{6}$ is ……… (a) $\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\sqrt{6}$ (d) $\frac{\sqrt{2}}{2}$
10	The additive inverse of the number $\left(\sqrt{7}-\sqrt{3}\right)$ is …………… (a) $\sqrt{7}+\sqrt{3}$ (b) $-\sqrt{7}+\sqrt{3}$ (c) $\sqrt{3}+\sqrt{7}$ (d) $-\sqrt{7}-\sqrt{3}$
11	The additive inverse of the number $\frac{6}{\sqrt{2}}$ is ……… (a) $-2\sqrt{3}$ (b) $2\sqrt{3}$ (c) $-3\sqrt{2}$ (d) $3\sqrt{2}$
12	The additive inverse of the number $(\sqrt{2}-\sqrt{5})$ is ……… (a) $\sqrt{2}+\sqrt{5}$ (b) $\sqrt{5}-\sqrt{2}$ (c) $\sqrt{2}-\sqrt{5}$ (d) $-\sqrt{2}-\sqrt{5}$
13	$\frac{1}{2}\sqrt{20}+10\sqrt{\frac{1}{5}}=$ ……… (a) $3\sqrt{5}$ (b) $4\sqrt{5}$ (c) 5 (d) 12
14	$\sqrt{3}\left(\sqrt{11}+\sqrt{3}\right)=$ ……… (a) $3\sqrt{11}+2$ (b) $\sqrt{33}+3$ (c) $11\sqrt{3}+2$ (d) $2\sqrt{11}+3$
15	$\sqrt{20}-\sqrt{5}=$ …………… (a) $\sqrt{15}$ (b) $\sqrt{5}$ (c) $3\sqrt{5}$ (d) 5
16	📖 $\sqrt{8}-\sqrt{2}=$ ……… (a) $\sqrt{6}$ (b) $\sqrt{2}$ (c) 2 (d) 1
17	📖 $(\sqrt{8}+\sqrt{2})^2=$ ……… (a) $\sqrt{10}$ (b) 10 (c) 18 (d) $\sqrt{18}$
18	📖 $(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})=$ ……… (a) 2 (b) 12 (c) $2\sqrt{7}$ (d) $-2\sqrt{5}$

[B] Essay Problems :

1	Reduce to the simplest form : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ **2016 Exam (7) Question (3) (a)**
2	Find the value of : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ **2016 Exam (5) Question (4) (a)**
3	Find the simplest form of : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ **2016 Exam (9) Question (3) (a)**
4	Find in simplest form the expression : $\sqrt{50}+2\sqrt{18}-\sqrt{32}$ **2016 Exam (11) Question (3) (a)**
5	Find in the simplest form : $\sqrt{50}+\sqrt{18}-\sqrt{2}$ **2016 Exam (12) Question (3) (a)**
6	Simplify to the simplest form : $\sqrt{32}+\sqrt{72}-6\sqrt{\frac{1}{2}}$ **2016 Exam (14) Question (3) (a)**
7	Simplify to the simplest form : $\sqrt{75}+\sqrt{48}-3\sqrt{3}$ **2016 Exam (15) Question (3) (a)**
8	If $X=[-2,3]$, $Y=[1,5[$, then find by using the number line : $X\cup Y$, $X-Y$ **2016 Exam (12) Question (3) (b)**
9	If $X=[-2,4]$ and $Y=]2,\infty[$, find each of the following using the number line : (1) $X\cap Y$ (2) $X-Y$ **2016 Exam (13) Question (3) (a)**
10	If $X=[-1,4]$, $Y=[2,7]$, then find each of the following by using the number line : (1) $X\cup Y$ (2) $X\cap Y$ (3) $X-Y$ **2016 Exam (14) Question (4) (b)**

LESSON (7) The two conjugate numbers

If a and b are two positive rational numbers , then each of the two numbers $(\sqrt{a}+\sqrt{b})$ and $(\sqrt{a}-\sqrt{b})$ is conjugate to the other one and we find that

- Their sum $= (\sqrt{a}+\sqrt{b}) + (\sqrt{a}-\sqrt{b}) = 2\sqrt{a}$ = twice the first term.
- Their product $= (\sqrt{a}+\sqrt{b})\ (\sqrt{a}-\sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$

  = the square of the first term – the square of the second term.

For example :

$(\sqrt{3}-\sqrt{2})$ its conjugate is $(\sqrt{3}+\sqrt{2})$, then we find that

- Their sum $= 2\sqrt{3}$
- Their product $= 3 - 2 = 1$

Remark

The product of the two conjugate numbers is always a rational number.

Remark

If we have a real number whose denominator is written in the form $(\sqrt{a}+\sqrt{b})$ or $(\sqrt{a}-\sqrt{b})$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

Important Remarks

From direct product (multiplying by inspection) ,

- **We know that :** $(x-y)(x+y) = x^2 - y^2$
- **And we know also :**

$(x+y)^2 = x^2 + 2xy + y^2$

Then

- $x^2 + xy + y^2 = (x+y)^2 - xy$
- $x^2 + y^2 = (x+y)^2 - 2xy$

$(x-y)^2 = x^2 - 2xy + y^2$

Then

- $x^2 - xy + y^2 = (x-y)^2 + xy$

or $x^2 + y^2 = (x-y)^2 + 2xy$

EXERCISES

[A] Complete the Following :

1	$(\sqrt{2}+\sqrt{3})^{-9}(\sqrt{2}-\sqrt{3})^{-9} =$ ....
2	$(\sqrt{2}+\sqrt{5})(\sqrt{2}-\sqrt{5}) =$ ....
3	$(\sqrt{3}+2)(\sqrt{3}-2) =$ ....
4	$(\sqrt{7}-\sqrt{2})(\sqrt{7}+\sqrt{2}) =$ ....
5	$(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3}) =$ ....
6	$(\sqrt{8}+\sqrt{2})(\sqrt{8}-\sqrt{2}) =$ ....
7	The conjugate number of the number $\frac{1}{\sqrt{3}-\sqrt{2}}$ is ....
8	The conjugate number of the number $1+\frac{7}{\sqrt{7}}$ in the simplest form is ....
9	The conjugate of the number $\sqrt{3}-5$ is ....
10	The conjugate number of the number : $\sqrt{5}+\sqrt{3}$ is ....
11	📖 If $x = 3+\sqrt{2}$, then its conjugate is .... and the product of multiplying x by its conjugate is ....
12	📖 The multiplicative inverse for $(\sqrt{3}+\sqrt{2})$ in its simplest form is ....
13	A rectangle of dimensions $(\sqrt{3}-1)$, $(\sqrt{3}+1)$ cm. has an area of .... cm^2
14	If $\frac{x}{5-\sqrt{5}} = 5+\sqrt{5}$, then the value of x in its simplest form is ....

15	If $\frac{1}{x} = \sqrt{5} - 2$, then the value of x in its simplest form is ………
16	If $x = \sqrt{3} + 2$, $y = \sqrt{3} - 2$, then $(xy , x + y) =$ ………
17	If $x = \sqrt[3]{3} + 1$ and $y = \sqrt[3]{3} - 1$, then $(x + y)^3 =$ ………
18	If : $x = 2\sqrt{3} + 3\sqrt{5}$ and $y = 2\sqrt{3} - 3\sqrt{5}$, then $x - y =$ ………
19	If : $x = \frac{1}{\sqrt{8} - \sqrt{5}}$ and $xy = \frac{1}{3}$, then $y =$ ………
20	If $x = \sqrt{3} + 1$, $y = \sqrt{3} - 1$, then $(x - y)^2 =$ ……………
21	If $x = 2 + \sqrt{5}$ and y is the conjugate number of x , then $(x - y)^2 =$ ………

[B] Essay Problems :

1	**Simplify :** (1) $(4 - 3\sqrt{2})(4 + 3\sqrt{2})$ (2) $(\sqrt{3} + 2)(\sqrt{3} - 1)$ **2016 Exam (1) Question (3) (a)**
2	If $x = \sqrt{5} + \sqrt{2}$, $y = \frac{3}{\sqrt{5} + \sqrt{2}}$ (1) **Prove that :** x and y are conjugate numbers. (2) **Find the value of :** $\frac{x + y}{xy}$ **2016 Exam (2) Question (3) (b)**
3	If $x = \sqrt{5} + \sqrt{3}$, $y = \frac{2}{\sqrt{5} + \sqrt{3}}$ **Find :** (1) $(x - y)^2$ (2) $(x + y)^2$ **2016 Exam (4) Question (4) (a)**
4	If $x = \sqrt{5} + \sqrt{3}$, $y = \sqrt{5} - \sqrt{3}$ **Find :** $x^2 - y^2$ **2016 Exam (8) Question (4) (a)**
5	If $x = \frac{1}{\sqrt{5} + 2}$, $y = \frac{20}{\sqrt{5}}$ **Find the value of :** $x^2 + y$ **2016 Exam (9) Question (3) (b)**

HOMEWORK

[A] Choose the correct answer:

1	The number $\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)$ is a .... number. (a) natural (b) rational (c) irrational (d) prime
2	The simplest form of the expression : $\left(\sqrt{3}-1\right)^2\left(\sqrt{3}+1\right)^2$ is .... (a) $2\left(\sqrt{3}-1\right)$ (b) $\left(\sqrt{3}+1\right)^2$ (c) 4 (d) 13
3	The multiplicative inverse of $\left(\sqrt{7}+\sqrt{3}\right)\left(\sqrt{7}-\sqrt{3}\right)$ is .... (a) 4 (b) -4 (c) $\frac{1}{4}$ (d) $\frac{-1}{4}$
4	If : $x=\sqrt{5}+\sqrt{3}$, $y=\sqrt{5}-\sqrt{3}$, then $x-y=$ .... (a) $2\sqrt{3}$ (b) $5\sqrt{3}$ (c) $2\sqrt{5}$ (d) 2
5	If : $x=3+\sqrt{5}$ and $y=3-\sqrt{5}$, then $x-y=$ .... (a) $6\sqrt{5}$ (b) $2\sqrt{5}$ (c) $\sqrt{10}$ (d) 6
6	If : $x^2-y^2=60$ and $x+y=5\sqrt{6}$, then $x-y=$ .... (a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) $3\sqrt{6}$ (d) $4\sqrt{6}$
7	If $x=3+\sqrt{3}$ and $y=3-\sqrt{3}$, then $x-y=$ .... (a) $6\sqrt{3}$ (b) $2\sqrt{3}$ (c) $\sqrt{6}$ (d) 6
8	If : $x=\sqrt{7}+\sqrt{3}$, $y=\sqrt{7}-\sqrt{3}$, then $(x-y)^3=$ .... (a) zero (b) 24 (c) $24\sqrt{3}$ (d) 196

[B] Essay Problems :

1

Simplify :

(1) $\left(4-3\sqrt{2}\right)\left(4+3\sqrt{2}\right)$ (2) $\left(\sqrt{3}+2\right)\left(\sqrt{3}-1\right)$

2016 Exam (1) Question (3) (a)

2

If $x=\sqrt{5}+\sqrt{2}$, $y=\dfrac{3}{\sqrt{5}+\sqrt{2}}$

(1) **Prove that :** x and y are conjugate numbers. (2) **Find the value of :** $\dfrac{x+y}{xy}$

2016 Exam (2) Question (3) (b)

3

If $a=\sqrt{2}+1$ and $b=\dfrac{1}{\sqrt{2}+1}$, **find the value of :** $(a-b)^2$

2016 Exam (3) Question (3) (a)

4

If $x=\sqrt{5}+\sqrt{3}$, $y=\dfrac{2}{\sqrt{5}+\sqrt{3}}$

Find : (1) $(x-y)^2$ (2) $(x+y)^2$

2016 Exam (4) Question (4) (a)

5

If $x=\sqrt{7}-\sqrt{3}$, $y=\sqrt{7}+\sqrt{3}$

Find : (1) xy (2) $(x+y)^2$

2016 Exam (5) Question (3) (a)

6

If $x=\sqrt{5}+\sqrt{2}$ and $y=\sqrt{5}-\sqrt{2}$, **find the value of :** $\dfrac{x+y}{xy-1}$

2016 Exam (7) Question (4) (a)

7

If $x=\sqrt{5}+\sqrt{3}$, $y=\sqrt{5}-\sqrt{3}$ **Find :** x^2-y^2

2016 Exam (8) Question (4) (a)

8

If $x=\dfrac{1}{\sqrt{5}+2}$, $y=\dfrac{20}{\sqrt{5}}$ **Find the value of :** x^2+y

2016 Exam (9) Question (3) (b)

9

If $x=\dfrac{4}{\sqrt{5}-\sqrt{3}}$, $y=\sqrt{20}-\sqrt{12}$

(1) **Find :** $x+y$ in the simplest form. (2) Are x and y conjugate numbers ? Why ?

2016 Exam (10) Question (4) (b)

10	If $x=\sqrt{3}+\sqrt{2}$, $y=\sqrt{3}-\sqrt{2}$, then find the value of : $\frac{x+y}{xy+1}$ **2016 Exam (11) Question (4) (b)**
11	If $x=\frac{1}{2+\sqrt{3}}$ and $y=\frac{12}{\sqrt{3}}$, find the value of : x^2+y in its simplest form. **2016 Exam (13) Question (3) (b)**
12	If $x=\sqrt{3}-\sqrt{2}$, $y=\frac{1}{\sqrt{3}-\sqrt{2}}$, prove that : x and y are conjugate numbers , then find the numerical value of $(x+y)^2$ **2016 Exam (14) Question (4) (a)**
13	Reduce to the simplest form : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ **2016 Exam (7) Question (3) (a)**
14	Find the value of : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ **2016 Exam (5) Question (4) (a)**
15	Find the simplest form of : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ **2016 Exam (9) Question (3) (a)**
16	Find in simplest form the expression : $\sqrt{50}+2\sqrt{18}-\sqrt{32}$ **2016 Exam (11) Question (3) (a)**
17	Find in the simplest form : $\sqrt{50}+\sqrt{18}-\sqrt{2}$ **2016 Exam (12) Question (3) (a)**
18	Simplify to the simplest form : $\sqrt{32}+\sqrt{72}-6\sqrt{\frac{1}{2}}$ **2016 Exam (14) Question (3) (a)**
19	Simplify to the simplest form : $\sqrt{75}+\sqrt{48}-3\sqrt{3}$ **2016 Exam (15) Question (3) (a)**

LESSON (8) Operations on the cube roots

If a and b are two real numbers , then

1 $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab}$

For example:

- $\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt[3]{3 \times 9} = \sqrt[3]{27} = 3$
- $\sqrt[3]{2} \times \sqrt[3]{-4} = \sqrt[3]{2 \times -4} = \sqrt[3]{-8} = -2$

2 $\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}}$ (where $b \neq 0$)

For example:

- $\frac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\frac{32}{4}} = \sqrt[3]{8} = 2$
- $\frac{\sqrt[3]{54}}{\sqrt[3]{-2}} = \sqrt[3]{\frac{54}{-2}} = \sqrt[3]{-27} = -3$

Remarks

* **If a and b are two real numbers , then :**

1 $\sqrt[3]{a^3 + b^3} \neq a + b$, $\sqrt[3]{a^3 - b^3} \neq a - b$

2 $\sqrt[3]{-a} = -\sqrt[3]{a}$

3 $a\sqrt[3]{b} = \sqrt[3]{a^3 b}$

For example :
- $3\sqrt[3]{\frac{1}{9}} = \sqrt[3]{27 \times \frac{1}{9}} = \sqrt[3]{3}$
- $8\sqrt[3]{\frac{1}{4}} = 4 \times 2\sqrt[3]{\frac{1}{4}} = 4\sqrt[3]{8 \times \frac{1}{4}} = 4\sqrt[3]{2}$

4 $\sqrt[3]{\frac{a}{b}} = \sqrt[3]{\frac{a}{b} \times \frac{b^2}{b^2}} = \sqrt[3]{\frac{a b^2}{b^3}} = \frac{1}{b}\sqrt[3]{a b^2}$

For example :
- $\sqrt[3]{\frac{1}{3}} = \sqrt[3]{\frac{1}{3} \times \frac{9}{9}} = \sqrt[3]{\frac{9}{27}} = \frac{1}{3}\sqrt[3]{9}$

Important Remark

$\sqrt[3]{16} = \sqrt[3]{8} \times \sqrt[3]{2} = 2\sqrt[3]{2}$

$\sqrt[3]{24} = \sqrt[3]{8} \times \sqrt[3]{3} = 2\sqrt[3]{3}$

$\sqrt[3]{54} = \sqrt[3]{27} \times \sqrt[3]{2} = 3\sqrt[3]{2}$

$\sqrt[3]{81} = \sqrt[3]{27} \times \sqrt[3]{3} = 3\sqrt[3]{3}$

$\sqrt[3]{128} = \sqrt[3]{64} \times \sqrt[3]{2} = 4\sqrt[3]{2}$

$\sqrt[3]{40} = \sqrt[3]{8} \times \sqrt[3]{5} = 2\sqrt[3]{5}$

$\sqrt[3]{250} = \sqrt[3]{125} \times \sqrt[3]{2} = 5\sqrt[3]{2}$

$\sqrt[3]{135} = \sqrt[3]{27} \times \sqrt[3]{5} = 3\sqrt[3]{5}$

EXERCISES

[A] Complete the Following :

1	$\sqrt[3]{2}+\sqrt[3]{2}=\sqrt[3]{\dots\dots\dots\dots}$
2	If : $x=\sqrt[3]{3}+7$, $y=\sqrt[3]{3}-7$, then $(x+y)^3=\dots\dots\dots$
3	If : $x=\sqrt[3]{3}+1$, $y=\sqrt[3]{3}-1$, then $(x+y)^3=\dots\dots\dots$
4	$\sqrt[3]{\frac{2}{3}}\times\sqrt[3]{-12}=\dots\dots\dots$
5	$\sqrt[3]{54}-\sqrt[3]{-16}=\sqrt[3]{\dots\dots\dots}$
6	The multiplicative neutral in $\mathbb{R}$ is .... and the additive neutral in $\mathbb{R}$ is ....
7	The additive inverse of the number $1-\sqrt{2}$ is ....
8	The multiplicative inverse of $\frac{2}{\sqrt{2}}$ is ....
9	If $a\in\mathbb{R}$ and $b\in\mathbb{R}$, then $a-b$ means the sum of the number a and .... of the number b
10	If $a\in\mathbb{N}$, $b\in\mathbb{Q}$ and $c\in\mathbb{R}$, then $a+b+c\in$.........
11	If : $a-b=2\sqrt{5}$, the value of : $a(a-b)^3+b(b-a)^3=\dots\dots\dots$
12	$\frac{3\sqrt{2}}{2\sqrt{18}}=\dots\dots\dots$
13	$\sqrt{3}\times\sqrt{6}=3\times\dots\dots\dots$
14	$\frac{1}{2}\sqrt{48}=2\times\dots\dots\dots$

[B] Essay Problems :

1	Find in simplest form : $\sqrt[3]{54}-\sqrt[3]{16}+\sqrt[3]{2}$ 2016 Exam (2) Question (4) (a)
2	Simplify : $\sqrt[3]{54}+\sqrt[3]{2}+\sqrt[3]{-128}$ 2016 Exam (4) Question (3) (b)
3	Simplify to the simplest form : $3\sqrt{12}+\sqrt[3]{54}-2\sqrt{27}-\sqrt[3]{16}$ 2016 Exam (6) Question (5) (a)
4	Prove that : $\sqrt[3]{128}+\sqrt[3]{16}-2\sqrt[3]{54}=0$ 2016 Exam (8) Question (3) (a)
5	Find the value of : $\sqrt{18}+\sqrt[3]{54}-3\sqrt{2}-\frac{1}{2}\sqrt[3]{16}$ 2016 Exam (10) Question (3) (a)
6	Simplify to the simplest form : $\sqrt[3]{16}-\frac{1}{3}\sqrt[3]{54}+\sqrt[3]{-2}$ 2016 Exam (13) Question (4) (a)

HOMEWORK

[A] Choose the correct answer:

1	$\sqrt[3]{(-8)^2} = \dots\dots\dots$ (a) 2 (b) -2 (c) 4 (d) -4
2	If : $x = \sqrt{3} + 2$ and $y = \sqrt{3} - 2$, then $(xy , x + y) = \dots\dots\dots$ (a) $(-1 , 2\sqrt{3})$ (b) $(1 , 2\sqrt{3})$ (c) $(5 , 2\sqrt{3})$ (d) $(-1 , 4)$
3	If : $x = \sqrt{7} + \sqrt{3}$, $y = \sqrt{7} - \sqrt{3}$, then $xy = \dots\dots\dots$ (a) 4 (b) 10 (c) 40 (d) 58
4	If : $x = \sqrt{5} + \sqrt{2}$, $y = \sqrt{5} - \sqrt{2}$, then $x - y = \dots\dots\dots$ (a) $2\sqrt{2}$ (b) $5\sqrt{2}$ (c) $2\sqrt{5}$ (d) 3
5	$\sqrt[3]{24} + \sqrt[3]{-81} + \sqrt[3]{3} = \dots\dots\dots$ (a) $\sqrt[3]{3}$ (b) 0 (c) $6\sqrt[3]{3}$ (d) $-\sqrt[3]{3}$
6	📖 $\sqrt[3]{54} + \sqrt[3]{-2} = \dots\dots\dots$ (a) $\sqrt[3]{52}$ (b) $\sqrt[3]{2}$ (c) $2\sqrt[3]{2}$ (d) $4\sqrt[3]{2}$
7	📖 $\sqrt[3]{-64} + \sqrt{16} = \dots\dots\dots$ (a) zero (b) 8 (c) -8 (d) ± 8
8	$\sqrt[3]{2} + \sqrt[3]{2} = \dots\dots\dots$ (a) $\sqrt[3]{2}$ (b) $\sqrt[3]{4}$ (c) $\sqrt[3]{8}$ (d) $\sqrt[3]{16}$
9	The number $(1 - \sqrt{3})(1 + \sqrt{3})$ is a $\dots\dots\dots\dots$ number. (a) natural (b) rational (c) irrational (d) prime

[B] Essay Problems :

No.	Problem
1	Find in simplest form : $\sqrt[3]{54}-\sqrt[3]{16}+\sqrt[3]{2}$ 2016 Exam (2) Question (4) (a)
2	Simplify : $\sqrt[3]{54}+\sqrt[3]{2}+\sqrt[3]{-128}$ 2016 Exam (4) Question (3) (b)
3	Simplify to the simplest form : $3\sqrt{12}+\sqrt[3]{54}-2\sqrt{27}-\sqrt[3]{16}$ 2016 Exam (6) Question (5) (a)
4	Prove that : $\sqrt[3]{128}+\sqrt[3]{16}-2\sqrt[3]{54}=0$ 2016 Exam (8) Question (3) (a)
5	Find the value of : $\sqrt{18}+\sqrt[3]{54}-3\sqrt{2}-\frac{1}{2}\sqrt[3]{16}$ 2016 Exam (10) Question (3) (a)
6	Simplify to the simplest form : $\sqrt[3]{16}-\frac{1}{3}\sqrt[3]{54}+\sqrt[3]{-2}$ 2016 Exam (13) Question (4) (a)
7	Reduce to the simplest form : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ 2016 Exam (7) Question (3) (a)
8	Find the value of : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ 2016 Exam (5) Question (4) (a)
9	Find the simplest form of : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$ 2016 Exam (9) Question (3) (a)
10	Find in simplest form the expression : $\sqrt{50}+2\sqrt{18}-\sqrt{32}$ 2016 Exam (11) Question (3) (a)
11	Find in the simplest form : $\sqrt{50}+\sqrt{18}-\sqrt{2}$ 2016 Exam (12) Question (3) (a)

LESSON (9) Applications on the real numbers

In the following , we will summarize the previous rules of areas and volumes of some solids :

The solid	The lateral area	The total area	The volume
The cube	$4\ell^2$	$6\ell^2$	ℓ^3
The cuboid	$2(x + y) \times z$	$2(xy + yz + zx)$	xyz
The cylinder	$2\pi r h$	$2\pi r h + 2\pi r^2$ $= 2\pi r(h + r)$	$\pi r^2 h$
The sphere	–	$4\pi r^2$	$\frac{4}{3}\pi r^3$

EXERCISES

[A] Complete the Following :

1	If the side length of a square is ℓ cm. and its area is 30 cm^2 , then the area of the square whose side length equals 2ℓ cm. is ………
2	Area of the square of side length is (2ℓ) cm. = ……… cm^2
3	The lateral area of a cube whose edge length is ℓ cm. = ……… cm^2.
4	📖 The edge length of a cube is 4 cm. , then its total area = ……… cm^2.

5	If the edge length of a cube is 5 cm. , then its volume = ... cm^3.
6	The cube whose edge length is 2 ℓ cm. , then its volume = ... cm^3.
7	If the volume of a cube is 216 cm^3. , then the length of its edge is ....
8	A cube whose volume is 1000 cm^3. , then its side length = ...
9	If the volume of a cube is 27 cm^3. , then its lateral area is .... cm^2.
10	The cube whose volume is ℓ^3 cm^3. , its total area = ... cm^2.
11	The sum of lengths of all edges of a cube is 36 cm. , then its total area equals .... cm^2.
12	The volume of a cuboid whose dimensions are $\sqrt{2}$ cm. , $\sqrt{3}$ cm. , $\sqrt{6}$ cm. is ....
13	The volume of right circular cylinder 90 π cm^3. and its height is 10 cm. then the radius length of its base equal .... cm.
14	The volume of the sphere = ...
15	The radius length of the sphere whose volume is $\frac{4}{3}$ π cm^3. = ...
16	The volume of the sphere whose diameter length is 6 cm. = ... π cm^3.
17	If the volume of a sphere is $\frac{9}{2}$ π cm^3. , then its radius length = ... cm.
18	If the volume of the sphere is $\frac{9}{16}$ π cm^3. , then its radius length = ... cm.

[B] Essay Problems :

1	The volume of a sphere is $\frac{99000}{7}$ cm^3. Calculate its radius length. $(\pi = \frac{22}{7})$ **2016 Exam (7) Question (5) (a)**

2	If the volume of a sphere is $\frac{32}{3}\pi$ cm^3, find the length of its diameter. **2016 Exam (1) Question (4) (b)**
3	Find the radius length of a sphere whose volume is 36π cm^3 **2016 Exam (2) Question (4) (b)**
4	If the volume of a right circular cylinder is 90π cm^3 and its height is 10 cm. find the radius length. **2016 Exam (3) Question (4) (b)**
5	A right circular cylinder , the radius length of its base is 3.5 cm. and its height 10 cm. Find the volume of the cylinder. $(\pi = 3.14)$ **2016 Exam (4) Question (4) (b)**
6	A metal cuboid with dimensions 77 cm. , 24 cm. , 21 cm. It was melted to make a sphere , find the radius length of that sphere. $(\pi = \frac{22}{7})$ **2016 Exam (6) Question (4) (b)**
7	Find the volume and the total area of a right circular cylinder in which the radius length of the base = 14 cm. and the height is 20 cm. **2016 Exam (8) Question (4) (b)**
8	The volume of the sphere is 36π cm^3 Find its radius length. **2016 Exam (9) Question (5) (a)**
9	If $\frac{3}{4}$ the volume of a sphere is 8π cm^3 , find its radius length. **2016 Exam (10) Question (5) (a)**
10	Find the surface area of a sphere if its radius length = 7 cm. $(\pi = \frac{22}{7})$ **2016 Exam (12) Question (4) (b)**
11	The right circular cylinder its radius length is 7 cm. and its height is 15 cm. **Find the lateral area.** $(\pi = \frac{22}{7})$ **2016 Exam (15) Question (4) (a)**

HOMEWORK

[A] Choose the correct answer:

1	The volume of a sphere which its diameter 6 cm. = ... cm^3 (a) 4π (b) 9π (c) 36π (d) 27π
2	If the radius length of a sphere is 3 cm. , then its volume is .... (a) 4π cm^3 (b) 9π cm^3 (c) 27π cm^3 (d) 36π cm^3
3	The volume of the sphere equals $32\sqrt{3}\pi$ cm^3 , then its radius length = ... (a) $\sqrt{3}$ cm. (b) 3 cm. (c) $2\sqrt{3}$ cm. (d) 9 cm.
4	If the volume of a sphere is $\frac{9}{16}\pi$ cm^3 , then its radius length is .... (a) $\frac{4}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$
5	If the volume of a sphere is $\frac{4}{3}\pi$ cm^3 , then its radius length = ... cm. (a) 2 (b) 1 (c) 8 (d) 27
6	The volume of a sphere is $\frac{9}{2}\pi$ cm^3 , then its radius length = ... cm. (a) 1 (b) 1.5 (c) 2 (d) 3
7	The lateral surface area of right circular cylinder = ... (a) $\pi r h$ (b) $4\pi r^2$ (c) $\pi r^2 h$ (d) $2\pi r h$
8	The radius length of a right circular cylinder whose volume is 40π cm^3 and its height 10 cm. = ... cm. (a) 5 (b) 3 (c) 2 (d) 1
9	The volume of a right circular cylinder is 90π cm^3 and its height is 10 cm. , then the radius length of its base = ... cm. (a) 3 (b) 4.5 (c) 5 (d) 9

No.	Question
10	Volume of a cube whose edge length 2ℓ cm. is .... cm^3 (a) $2\ell^3$ (b) 8ℓ (c) $8\ell^3$ (d) ℓ^3
11	The volume of a cube whose edge length is 2 cm. is .... cm^3 (a) 8 (b) 4 (c) 16 (d) 6
12	If the area of one face of a cube is 25 cm^2 , then it's volume = ... (a) 125 cm^3 (b) 50 cm^2 (c) 5 cm^3 (d) 625 cm^3
13	The edge length of a cube whose volume is $2\sqrt{2}$ cm^3 = ... cm. (a) $\sqrt{2}$ (b) 2 (c) 8 (d) 1.5
14	The cube whose volume is $3\sqrt{3}$ cm^3 , its side length is .... cm. (a) 3 (b) $\sqrt{3}$ (c) 9 (d) 27
15	The edge length of a cube whose volume is 3 m^3 is .... cm. (a) $\sqrt{3}$ (b) 3 (c) 1 (d) $\sqrt[3]{3}$
16	The cube whose volume is 8 cm^3 , then its total area = ... cm^2 (a) 16 (b) 24 (c) 96 (d) 4
17	If the volume of a cube is 27 cm^3 , then the total area is .... cm^2 (a) 54 (b) 9 (c) 27 (d) 36
18	If the volume of a cube is $40\sqrt{5}$ cm^3 , then its edge length is .... cm. (a) $\sqrt{5}$ (b) $8\sqrt{5}$ (c) $2\sqrt{5}$ (d) $5\sqrt{2}$
19	The volume of a cube is 64 cm^3 , then its total area = ... (a) 64 cm^2 (b) 96 cm^2 (c) 36 cm^2 (d) 24 cm^2
20	A cube of volume 216 cm^3 has a total area = ... cm^2 (a) 6 (b) 36 (c) 144 (d) 216

[B] Essay Problems :

1	The volume of a sphere is $\frac{99000}{7}$ cm³ Calculate its radius length. $(\pi = \frac{22}{7})$ **2016 Exam (7) Question (5) (a)**
2	If the volume of a sphere is $\frac{32}{3}\pi$ cm³ , find the length of its diameter. **2016 Exam (1) Question (4) (b)**
3	Find the radius length of a sphere whose volume is 36π cm³ **2016 Exam (2) Question (4) (b)**
4	If the volume of a right circular cylinder is 90π cm³ and its height is 10 cm. find the radius length. **2016 Exam (3) Question (4) (b)**
5	A right circular cylinder , the radius length of its base is 3.5 cm. and its height 10 cm. Find the volume of the cylinder. $(\pi = 3.14)$ **2016 Exam (4) Question (4) (b)**
6	A metal cuboid with dimensions 77 cm. , 24 cm. , 21 cm. It was melted to make a sphere , find the radius length of that sphere. $(\pi = \frac{22}{7})$ **2016 Exam (6) Question (4) (b)**
7	Find the volume and the total area of a right circular cylinder in which the radius length of the base = 14 cm. and the height is 20 cm. **2016 Exam (8) Question (4) (b)**
8	The volume of the sphere is 36π cm³ Find its radius length. **2016 Exam (9) Question (5) (a)**
9	If $\frac{3}{4}$ the volume of a sphere is 8π cm³ , find its radius length. **2016 Exam (10) Question (5) (a)**
10	Find the surface area of a sphere if its radius length = 7 cm. $(\pi = \frac{22}{7})$ **2016 Exam (12) Question (4) (b)**
11	The right circular cylinder its radius length is 7 cm. and its height is 15 cm. **Find the lateral area.** $(\pi = \frac{22}{7})$ **2016 Exam (15) Question (4) (a)**

LESSON (10) Solving equations and ikequalities Of first degree in one variable in R

[A] Find the S.S of each of the following equations in R :

1) $2X = 10$	2) $X - 3 = 5$	3) $2X - 3 = 11$	4) $3X + 1 = 13$
$X = 10 \div 2$ $X = 5$ $S.S = \{5\}$	$X = 5 + 3$ $X = 8$ $S.S = \{8\}$	$X = \frac{11+3}{2}$ $X = 7$ $S.S = \{7\}$	$X = \frac{13-1}{3}$ $X = 4$ $S.S = \{4\}$
5) $X + 5 \geq 8$	6) $2X > 10$	7) $4X + 1 \leq 21$	8) $5X - 4 < 26$
$X \geq 8 - 5$ $X \geq 3$ $[3, \infty[$	$X > 10 \div 2$ $X > 5$ $]5, \infty[$	$X \leq \frac{21-1}{4}$ $X \leq 5$ $]-\infty, 5]$	$X < \frac{26+4}{5}$ $X < 6$ $]-\infty, 6[$
9) $3 < X + 1 \leq 5$	10) $7 < 3X + 1 \leq 22$	11) $5 \leq 3 - 2X \leq 7$	12) $-2 \leq 3X + 7 < 22$
$3 - 1 < X \leq 5 - 1$ $2 < X \leq 4$ $]2, 4]$	$\frac{7-1}{3} < X \leq \frac{22-1}{3}$ $2 < X \leq 7$ $]2, 7]$	$\frac{5-3}{-2} \geq X \geq \frac{7-3}{-2}$ $-1 \geq X \geq -2$ $[-2, -1]$	$\frac{-2-7}{3} \leq X < \frac{22-7}{3}$ $-3 \leq X < 5$ $[-3, 5[$
13) $3 + X < 2X < 7 + X$	14) $3 + X \leq 2X + 1 < 7 + X$	15) $3 - X < 2X \leq 12 - X$	16) $14 - X \leq 5X + 2 \leq 26 - X$
$3 < 2X - X < 7$ $3 < X < 7$ $]3, 7[$	$3 \leq 2X - X + 1 < 7$ $3 \leq X + 1 < 7$ $3 - 1 \leq X < 7 - 1$ $2 \leq X < 6$ $[2, 6[$	$3 < 2X + X \leq 12$ $3 < 3X \leq 12$ $3 \div 3 < 3X \leq 12 \div 3$ $1 < X \leq 4$ $]1, 4]$	$14 \leq 5X + X + 2 \leq 26$ $14 \leq 6X + 2 \leq 26$ $\frac{14-2}{6} \leq X \leq \frac{26-2}{6}$ $2 \leq X \leq 4$ $[2, 4]$

EXERCISES

[A] Complete the Following :

1	The set $X = \{x : x \geq 2 , x \in \mathbb{R}\}$ in the form of an interval is ....
2	The solution set of the equation : $x + \sqrt{2} = \sqrt{8}$ in $\mathbb{R}$ is ....
3	If : $x + 2\sqrt{3} = 3$, then $x =$ ....
4	The S.S. of the equation $\sqrt{2}\, x - 1 = 1$ is .... where $x \in \mathbb{R}$
5	The S.S. of the inequality : $-x + 1 \leq 0$ in $\mathbb{R}$ is ....
6	If $x - 3 \geq 0$, then x ....
7	If $5x < 15$, then x ....
8	If $1 - x > 4$, then x ....
9	If $-2x \leq 3$, then x ....
10	If $\sqrt{2}\, x \leq 4$, then x ....
11	The S.S. of the inequality : $4 < 2x < 8$ in $\mathbb{R}$ is ....
12	The S.S. of the inequality : $-5 \leq -x < 2$ in $\mathbb{R}$ is ....
13	The S.S. of the inequality : $2 - x < 0$ in $\mathbb{R}$ is ....
14	If $-3 < x < 3$ where $x \in \mathbb{R}$, then $2x \in]-6$, ... [
15	If $-x < 2$, then $x \in$ ....
16	The S.S. of the inequality : $-x + 1 \leq 0$ in $\mathbb{R}$ is ....

17	If $2 < x < 5$, then $3x - 1 \in$ ....
18	If : $x < -\sqrt{7} < x + 1$, then $x =$ .... (where x is an integer)
19	If : $x < \sqrt{15} < x + 1$, $x \in \mathbb{Z}$, then $x =$ ....
20	If : $x < \sqrt{19} < x + 1$, then $x =$ ....
21	$\mathbb{Q} \cap \grave{\mathbb{Q}} =$ ....
22	$\mathbb{Q} \cup \grave{\mathbb{Q}} =$ ....
23	$\mathbb{R}_+ \cup \mathbb{R}_- =$ ....
24	The multiplicative inverse of $\frac{\sqrt{5}}{10}$ is ....
25	The multiplicative inverse of the number : $(\sqrt{3} + \sqrt{2})$ is ....
26	$\sqrt[3]{64 + \text{..........}} = 5$
27	$\sqrt[3]{\text{..........}} = 4$
28	$\sqrt{16} = \sqrt[3]{\text{..........}}$
29	$\sqrt[3]{64} = \sqrt{\text{..........}}$
30	If : $\sqrt[3]{64} = \sqrt{x}$, then $2x =$ ....
31	The solution set of the equation : $x^2 + 4 = 0$ in $\mathbb{R}$ is ....
32	The solution set of the equation : $x^2 + 9 = 0$ in $\mathbb{Q}$ is ....
33	The S.S. of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is ....

[B] Essay Problems :

1	**Find in $\mathbb{R}$ the S.S. of :** (1) $5x + 6 = 1$ (2) $3 < x + 2 \le 6$ **2016 Exam (1) Question (3) (b)**
2	**Find in $\mathbb{R}$ the S.S. of :** $4 < 3x + 1 \le 10$ and represent it on the number line. **2016 Exam (2) Question (3) (a)**
3	**Find the S.S. of the inequality :** $-2 < 3x + 7 < 10$ in $\mathbb{R}$, then represent it on the number line. **2016 Exam (5) Question (3) (b)**
4	**Find the S.S. of the inequality :** $-3 \le 2x - 1 < 5$ in $\mathbb{R}$ by using number line **2016 Exam (6) Question (3) (a)**
5	**Find in $\mathbb{R}$ the S.S. of the inequality :** $-2 < 3x + 7 \le 10$ **2016 Exam (7) Question (4) (b)**
6	**Find the S.S. in $\mathbb{R}$:** $5 \le 3 - 2x \le 7$ **2016 Exam (8) Question (3) (b)**
7	**Find the S.S. of the inequality :** $-2 < 3x + 7 \le 10$ in $\mathbb{R}$, then represent it on the number line. **2016 Exam (9) Question (4) (a)**
8	**Find the S.S. of the inequality :** $x - 5 < 2x + 4 \le x + 3$ in $\mathbb{R}$, then represent the interval of the solution on the number line. **2016 Exam (11) Question (3) (b)**
9	**Find the S.S. of the inequality :** $3x - 5 \ge 7$ in $\mathbb{R}$, then represent the interval of the solution on the number line. **2016 Exam (12) Question (4) (a)**
10	**Find the solution set for the inequality :** $2x + 5 \ge 3$ in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line. **2016 Exam (13) Question (4) (b)**

HOMEWORK

[A] Choose the correct answer:

1	The S.S. of the inequality : $0 < x + 5 \le 6$ in $\mathbb{R}$ is .... (a) $]5 , 11]$ (b) $]-1 , 5]$ (c) $[-5 , 1[$ (d) $]-5 , 1]$
2	The S.S. of the inequality : $-x > 2$ in $\mathbb{R}$ is .... (a) $\{2\}$ (b) $]-\infty , 2[$ (c) $]2 , \infty[$ (d) $]-\infty , -2[$
3	If $-1 < -x \le 5$, then the S.S. in $\mathbb{R}$ is .... (a) $[-5 , 1[$ (b) $[5 , -1[$ (c) $]-5 , 1]$ (d) $]-5 , 1[$
4	$[-3 , 7] - \{-3 , 7\} =$ .... (a) $[-3 , 7[$ (b) $]-3 , 7]$ (c) $]-3 , 7[$ (d) $[-3 , 7]$
5	The S.S. of the inequality : $-x > 3$ in $\mathbb{R}$ is .... (a) $\{3\}$ (b) $]3 , \infty[$ (c) $]-\infty , 3[$ (d) $]-\infty , -3[$
6	The S.S. of equation : $\sqrt{2}\, x = 2$ in $\mathbb{R}$ is .... (a) $\{\sqrt{2}\}$ (b) $\sqrt{2}$ (c) $\{2\}$ (d) $\{2\sqrt{2}\}$
7	$\{x : x \in \mathbb{R} , x < 1\} =$ .... (a) $0 , -1 , -2 , \cdots$ (b) $]-\infty , 1]$ (c) $]-\infty , 1[$ (d) $]-\infty , 0]$
8	If : $x \in \mathbb{R}$, $1 - 7x > \lvert -8 \rvert$, then $x <$ .... (a) 1 (b) -1 (c) $\frac{9}{7}$ (d) 0
9	If : $2 < x < 5$, then $3x - 1 \in$ .... (a) $]3 , 12[$ (b) $]6 , 14[$ (c) $]5 , 15[$ (d) $]5 , 14[$

10 If : $-2x > -6$, then $x \in$ ....

(a) $]-\infty , 3[$ (b) $]3 , \infty[$ (c) $]-2 , -6[$ (d) $]1 , 3[$

11 If : $-2x > -6$, then $x \in$ ....

(a) $]-\infty , 3[$ (b) $]3 , \infty[$ (c) $]-2 , -6[$ (d) $]1 , 3[$

12 The solution set in $\mathbb{R}$ of the inequality : $-x < 3$ is ....

(a) $]-\infty , 3[$ (b) $]3 , \infty[$ (c) $]-\infty , -3[$ (d) $]-3 , \infty[$

13 The S.S. of the inequality : $-x > 2$ in $\mathbb{R}$ is ....

(a) $]-\infty , 2[$ (b) $]2 , \infty[$ (c) $]-\infty , -2[$ (d) $\{2\}$

14 The S.S. of the inequality : $-x > 3$ in $\mathbb{R}$ is ....

(a) $\{3\}$ (b) $]3 , \infty[$ (c) $]-\infty , 3[$ (d) $]-\infty , -3[$

15 If the S.S of the inequality : $-1 < x + 3 < 3$ in $\mathbb{R}$ is ....

(a) $[-4 , 0]$ (b) $[2 , 6]$ (c) $]2 , 6[$ (d) $]-4 , 0[$

16 The S.S. of the inequality : $-x > 3$ in $\mathbb{R}$ is ....

(a) $\{-3\}$ (b) $]3 , \infty[$ (c) $]-\infty , 3[$ (d) $]-\infty , -3[$

17 $\left(2\sqrt[3]{2}\right)^3 =$ ....

(a) 4 (b) 8 (c) 16 (d) 40

18 $\sqrt[3]{(-8)^2} =$ ....

(a) 2 (b) -2 (c) 4 (d) -4

19 $\sqrt{8} - \sqrt{2} =$ ....

(a) $\sqrt{6}$ (b) 2 (c) $\sqrt{2}$ (d) 1

20	$\sqrt{25} - \sqrt[3]{-125} = \dots\dots$ (a) 10 (b) zero (c) 5 (d) ± 5
21	$-2\sqrt{3} \times \sqrt{3} = \dots\dots$ (a) $-2\sqrt{3}$ (b) -6 (c) $2\sqrt{3}$ (d) 6
22	$\sqrt{3}\left(\sqrt{11}+\sqrt{3}\right) = \dots\dots$ (a) $3\sqrt{11}+2$ (b) $\sqrt{33}+3$ (c) $11\sqrt{3}+2$ (d) $2\sqrt{11}+3$
23	$\sqrt{9} + \sqrt[3]{-27} = \dots\dots$ (a) 0 (b) -6 (c) -9 (d) ± 6
24	$\sqrt[3]{-8} + \sqrt{4} = \dots\dots$ (a) 4 (b) -4 (c) zero (d) 8
25	$\sqrt{25} = \sqrt[3]{\dots\dots}$ (a) 5 (b) 15 (c) 125 (d) -5
26	If : $\sqrt[3]{y} = -\sqrt{9}$, then y = $\dots\dots$ (a) 3 (b) -3 (c) -27 (d) 27
27	$\sqrt{25} + \sqrt[3]{-27} = \sqrt{\dots\dots}$ (a) 8 (b) 4 (c) 2 (d) 5
28	$\sqrt[3]{27} = \sqrt{x+3}$, then $x = \dots\dots$ (a) 3 (b) 6 (c) 9 (d) 12
29	If $x^3 = 64$, then $\sqrt{x} = \dots\dots$ (a) 4 (b) -4 (c) 2 (d) -2

30	The solution set for the equation : $x^2 = 2$ in $\mathbb{R}$ is ……… (a) $\{\sqrt{2}\}$ (b) $\{-\sqrt{2}\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\{2\}$
31	The S.S. of the equation : $x^2 + 3 = 0$ in $\mathbb{R}$ is ……… (a) $\varnothing$ (b) $-\sqrt{3}$ (c) $\sqrt{3}$ (d) $\pm\sqrt{3}$
32	The S.S. of the equation : $x^2 + 5 = 9$ where $x \in \mathbb{Q}$ is …………… (a) $\{4\}$ (b) $\{-2, 2\}$ (c) $\varnothing$ (d) $\{13\}$
33	The S.S. of the equation : $x^3 + 8 = 0$ in $\mathbb{R}$ is …………… (a) $\{2\}$ (b) $\{2\sqrt{2}\}$ (c) $\{-2\}$ (d) $\{2, -2\}$
34	The solution set for the equation : $x^3 + 9 = 8$ in $\mathbb{R}$ is ……… (a) $\{8\}$ (b) $\{9\}$ (c) $\{3\}$ (d) $\{-1\}$
35	The S.S. of the equation : $x^3 + 27 = 0$ in $\mathbb{R}$ is …………… (a) $\{3\}$ (b) $\{-3\}$ (c) $\{3\sqrt{3}\}$ (d) $\{\pm 3\sqrt{3}\}$
36	The S.S. in $\mathbb{R}$ of the equation : $x^3 + 11 = 12$ in $\mathbb{R}$ is …………… (a) $\{11\}$ (b) $\{12\}$ (c) $\{1\}$ (d) $\{3\}$

[B] Essay Problems :

1	**Find in $\mathbb{R}$ the S.S. of :** (1) $5x + 6 = 1$ (2) $3 < x + 2 \le 6$ **2016 Exam (1) Question (3) (b)**
2	**Find in $\mathbb{R}$ the S.S. of :** $4 < 3x + 1 \le 10$ and represent it on the number line. **2016 Exam (2) Question (3) (a)**
3	**Find the S.S. of the inequality :** $-2 < 3x + 7 < 10$ in $\mathbb{R}$, then represent it on the number line. **2016 Exam (5) Question (3) (b)**

4	Find the S.S. of the inequality : $-3 \leq 2x - 1 < 5$ in $\mathbb{R}$ by using number line **2016 Exam (6) Question (3) (a)**
5	Find in $\mathbb{R}$ the S.S. of the inequality : $-2 < 3x + 7 \leq 10$ **2016 Exam (7) Question (4) (b)**
6	Find the S.S. in $\mathbb{R}$: $5 \leq 3 - 2x \leq 7$ **2016 Exam (8) Question (3) (b)**
7	Find the S.S. of the inequality : $-2 < 3x + 7 \leq 10$ in $\mathbb{R}$, then represent it on the number line. **2016 Exam (9) Question (4) (a)**
8	Find the S.S. of the inequality : $x - 5 < 2x + 4 \leq x + 3$ in $\mathbb{R}$, then represent the interval of the solution on the number line. **2016 Exam (11) Question (3) (b)**
9	Find the S.S. of the inequality : $3x - 5 \geq 7$ in $\mathbb{R}$, then represent the interval of the solution on the number line. **2016 Exam (12) Question (4) (a)**
10	Find the solution set for the inequality : $2x + 5 \geq 3$ in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line. **2016 Exam (13) Question (4) (b)**
11	Find in $\mathbb{R}$ the S.S. of the inequality : $4 < 3x + 1 \leq 10$ and represent it on the number line. **2016 Exam (14) Question (3) (b)**
12	Find the S.S. of the inequality : $1 < 2x + 1 < 5$ in $\mathbb{R}$, then represent the interval of the solution on the number line. **2016 Exam (15) Question (5) (a)**

Algebra

Relationships & Statistics

SECOND PREP

First Term 2019

UNIT 2 LESSON (1) relation between two variables

The linear relation

- It is a relation of the first degree between two variables x and y , it is in the form $ax + by = c$, where a , b and c are real numbers , a and b are not both equal to zero
- There is an infinite number of ordered pairs which satisfy this relation.
- If we represent it graphically , the graph will be a straight line therefore it is called a linear relation , this will be shown later when we study the graphic representation of the linear relation.

The graphic representation of the linear relation

Example 5 **Represent the relation :** $2x - y = 3$ graphically

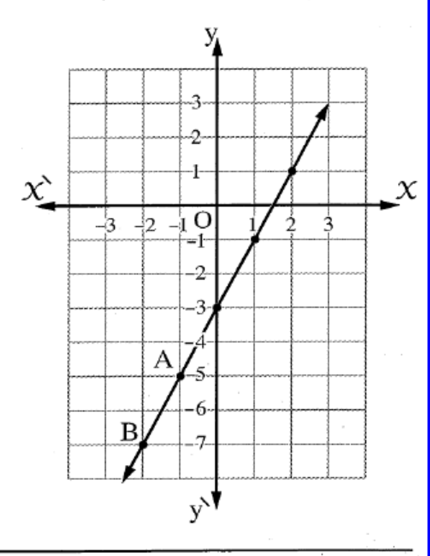
Solution

To represent this relation graphically , we should determine three ordered pairs satisfying the relation : $2x - y = 3$, as follows :

- Set $x = 0$ $\therefore 2 \times 0 - y = 3$ $\therefore -y = 3$ $\therefore y = -3$
- Set $x = 1$ $\therefore 2 \times 1 - y = 3$ $\therefore -y = 1$ $\therefore y = -1$
- Set $x = 2$ $\therefore 2 \times 2 - y = 3$ $\therefore -y = -1$ $\therefore y = 1$

It is preferable to put the values of x and y in a table as the following :

x	0	1	2
y	– 3	– 1	1



Then we determine the points which represent these ordered pairs : (0 , – 3) , (1 , – 1) and (2 , 1) on orthogonal coordinates system , then we draw the straight line passing through these points , it will be the graphic representation of the relation : $2x - y = 3$

Remark

All the points of the straight line which represents the relation determine ordered pairs which satisfy the relation.

For example:

The point A determines the ordered pair (– 1 , – 5) which satisfies the relation when we put $x = -1$ we find that $2 \times (-1) - y = 3$ **i.e.** $y = -5$ and also the point B (– 2 , – 7)

Special cases

We studied before the relation : a x + b y = c , where a , b are not both equal to zero and it is called a linear relation and it is represented graphically by a straight line and now we study the following cases :

1 If a = 0 , b ≠ 0

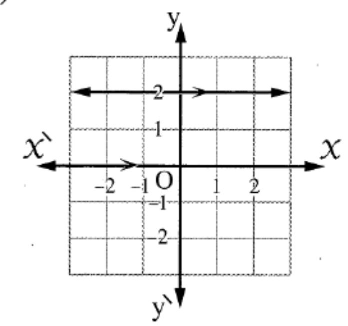
Then the relation becomes in the form :

b y = c

and it is represented graphically by a straight line parallel to x-axis and intersects y-axis at the point $\left(0 , \frac{c}{b}\right)$

For example:

The relation : 2 y = 4 i.e. y = 2 is represented by a straight line parallel to x-axis and intersects y-axis at the point (0 , 2)



Notice that :

The relation : y = 0 is represented by x-axis

2 If b = 0 , a ≠ 0

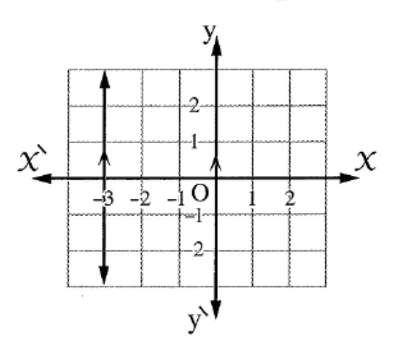
Then the relation becomes in the form :

a x = c

and it is represented graphically by a straight line parallel to y-axis and intersects x-axis at the point $\left(\frac{c}{a} , 0\right)$

For example:

The relation : $x = -3$ is represented by a straight line parallel to y-axis and intersects x-axis at the point (– 3 , 0)



Notice that :

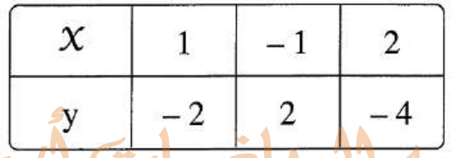
The relation : x = 0 is represented by y-axis

3 If c = 0

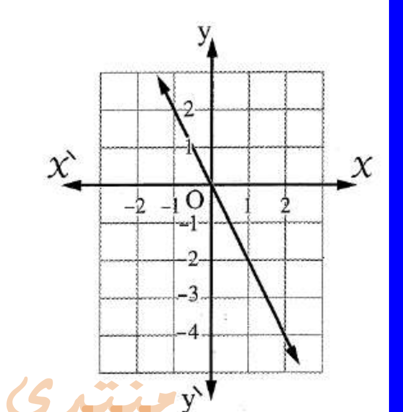
Then the relation becomes : **a x + b y = 0** and it is represented by a straight line passing through the origin point.

For example:

The relation : 2 x + y = 0 is represented graphically by a straight line passing through the origin point as shown in the opposite graph :



x	1	– 1	2
y	– 2	2	– 4



UNIT 2 LESSON (2) Slope of straight line

If A and B are two points in the coordinates plane where A (x_1, y_1) and B (x_2, y_2), then :

The slope of the straight line $\overleftrightarrow{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_1 \neq x_2$

Definition

$$\text{The slope of the straight line} = \frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$$

i.e. • $S = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$ • S is undefined if $x_1 = x_2$

Notice that

The straight line passes through the two points (2 , 0) and (7 , 5) , then :

the slope of the straight line $L = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-0}{7-2} = \frac{5}{5} = 1$

Remark

The angle which the straight line L makes with the positive direction of the x-axis takes one of the following cases :

1 Acute angle	2 Obtuse angle	3 Zero angle	4 Right angle
The slope is positive	The slope is negative	The slope is zero	The slope is undefined

Example 5 **Prove that the points A (2 , 3) , B (4 , 2) and C (8 , 0) are collinear.**

Solution

$\because S = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ The slope of $\overleftrightarrow{AB} = \frac{2-3}{4-2} = -\frac{1}{2}$, the slope of $\overleftrightarrow{BC} = \frac{0-2}{8-4} = \frac{-2}{4} = -\frac{1}{2}$

, $\because$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is common.

$\therefore$ The points A , B and C are collinear.

EXERCISES

[A] Choose the correct answer:

1	Which of the following represent linear relation ? A) $Xy = 2$ B) $X^2 = \frac{1}{y}$ C) $\frac{X}{y} = 1$ D) $y = X^2 + 4$	
2	Which of the following satisfies the relation : $2X + y = 5$? A) (– 3 , 3) B) (1 , 3) C) (3 , 1) D) (2 , 2)	
3	(3 , 2) satisfies the relation .... A) $Y + X = 5$ B) $Y – X = 5$ C) $3Y – X = 2$ D) $2X + Y = 1$	
4	(3 , 2) does not satisfy the relation .... A) $Y + X = 5$ B) $X – Y = 1$ C) $Y + X = 7$ D) $3Y – X = 3$	
5	Value of b where (– 3 , 2) satisfies the relation : $3X + by = 1$ is A) 3 B) 5 C) 4 D) 0	
6	If : (2 , - 5) satisfies the relation : $3X – Y + c = 0$, then c = ... A) 1 B) – 1 C) 11 D) – 11	
7	If : (– 1 , 5) satisfies the relation : $3X + ky = 7$, then k = ... A) 2 B) – 2 C) 1 D) 10	
8	If : (a , 1) satisfies the relation : $2X + 3y = 7$, then a = ... A) 2 B) – 2 C) 4 D) 3	
9	If : (2 , b) satisfies the relation : $3X + y = 9$, then b = ... A) 6 B) 3 C) 2 D) 0	
10	If : (a , 4) satisfies the relation : $X – y = -1$, then a = ... A) $\sqrt{3}$ B) 3 C) 27 D) 1.5	
11	If : (a , 2 a) satisfies the relation : $y = X - 1$, then a = ... A) 1 B) 10 C) - 1 D) 3	
12	If : (k , 2 k) satisfies the relation : $3X + 2y = 14$, then k = ... A) 2 B) - 2 C) 7 D) 0	

13 If : (2 k , 3 k) satisfies the relation : X + y = 15 , then k = ...
A) 5 B) 3 C) - 5 D) - 3

14 If : (2 m , m) satisfies the relation : 2 X + 3 y = 35 , then m = ...
A) 7 B) 5 C) 14 D) 10

15 The opposite table shows the relation between x and y , which is ....

x	1	2	3	4	5
y	1	3	5	7	9

(a) $y = x + 4$ (b) $y = x + 1$
(c) $y = 2x - 1$ (d) $y = 3x - 2$

16 The slope of the straight line parallel to the X – axis is ....
A) Positive B) Negative C) Zero D) Undefined

17 The slope of the straight line parallel to the Y – axis is ....
A) Positive B) Negative C) Zero D) Undefined

18 The slope of horizontal line is ....
A) 1 B) Zero C) - 1 D) Undefined

19 If A (3 , 2) , B (0 , 4) , then the slope of $\overleftrightarrow{AB}$ = ...
A) - 2 B) 2 C) $\frac{1}{2}$ D) $-\frac{1}{2}$

20 Slope of straight line passes through (- 2 , 3) and (2 , 3) is ....
A) 2 B) 1 C) Zero D) Undefined

21 Slope of straight line passes through (- 3 , 1) and (2 , 5) is ....
A) $\frac{4}{5}$ B) $-\frac{6}{1}$ C) $\frac{5}{4}$ D) $-\frac{1}{6}$

22 Slope of straight line passes through (3 , 2) and (– 5 , 3) is ....
A) $\frac{1}{8}$ B) $-\frac{1}{8}$ C) 8 D) – 8

23 If A (3 , 5) , B (5 , – 1) , then the slope of $\overleftrightarrow{AB}$ = ...
A) – 3 B) $-\frac{1}{3}$ C) 3 D) $\frac{1}{3}$

24 Slope of straight line passes through (3 , 8) and (0 , 2) is ....
A) 1 B) 2 C) 1 D) $\frac{1}{2}$

25 Slope of straight line passes through (2 , 3) and (4 , 7) is ....

A) 3　　B) 2　　C) 1　　D) $\frac{1}{2}$

26 Slope of straight line passes through (3 , y) and (5 , – 2) is – 3 , then y = ...

A) 2　　B) 4　　C) 6　　D) – 30

27 Slope of straight line passes through (2 , 6) and (7 , 11) is ....

A) 1　　B) 2　　C) 5　　D) 6

28 If the Slope of straight line a X + b y + 1 = 0 is undefined , then ....

A) a = b　　B) a = zero　　C) b = zero　　D) a = – b

29 Relation : X – 5 = 0 is represented by a st. line whose slope is............

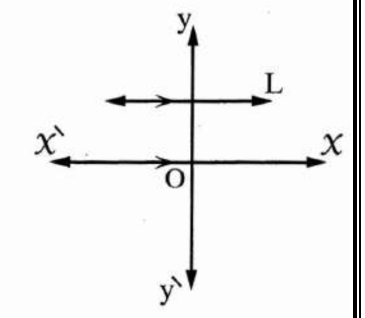
A) 0　　B) – 5　　C) 5　　D) Undefined

30 **In the opposite figure :**

The slope of the straight line L is ....

(a) positive.　　(b) negative.

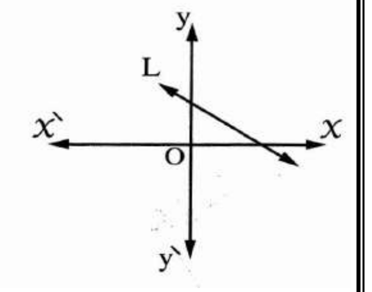
(c) zero.　　(d) undefined.



31 The slope of the straight line L in the opposite figure is ....

(a) positive.　　(b) negative.

(c) zero.　　(d) undefined.

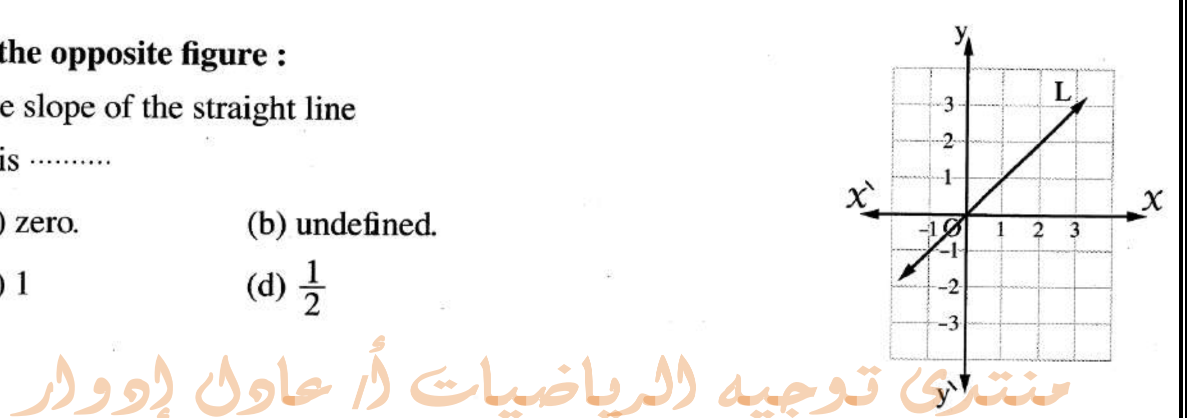


32 **In the opposite figure :**

The slope of the straight line L is ....

(a) zero.　　(b) undefined.

(c) 1　　(d) $\frac{1}{2}$



[B] Essay Problems :

1	**Represent graphically the relation :** $y + 2x = 5$ **2016 Exam (4) Question (5) (a)**
2	**Graph the relation :** $2x + 3y = 6$ Find the intersection points with x-axis and y-axis. **2016 Exam (6) Question (3) (b)**
3	If (a , 2 a) satisfies $y = x - 1$, **find the value of :** a **2016 Exam (3) Question (3) (b)**
4	**Find the value of :** k where (k , 2 k) satisfies the relation : $x + y = 15$ **2016 Exam (11) Question (4) (a)**
5	If (a , 3) satisfies the relation : $y = 2x - 1$ **find the value of a** **2016 Exam (15) Question (3) (b)**
6	**Using the linear relations , complete the following tables :** (1) $4x - y = -1$ x: 0, 1, 2, 3 / y: ...., ..., ..., ... (2) $y = 5x + 15$ x: −4, −3, −2 / y: ...., ..., ... **Exercise (11) Question (4)**
7	Graph the relation : $2x + 3y = 6$ If the straight line representing this relation intersects the x-axis at point A and the y-axis at point B , find the area of the triangle OAB where O is the origin point. « 3 square units » **Exercise (11) Question (13)**

(1) $4x - y = -1$

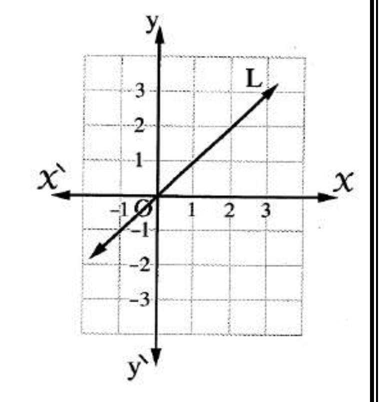
x	0	1	2	3
y	....	....	....	....

(2) $y = 5x + 15$

x	−4	−3	−2
y	....	....	....

HOMEWORK

[A] Choose the correct answer:

1	Which of the following satisfies the relation : $2X + y = 5$? A) (– 3 , 3) B) (1 , 3) C) (3 , 1) D) (2 , 2)	
2	If : (– 1 , 5) satisfies the relation : $3X + ky = 7$, then k = ... A) 2 B) – 2 C) 1 D) 10	
3	If : (k , 2 k) satisfies the relation : $3X + 2y = 14$, then k = ... A) 2 B) - 2 C) 7 D) 0	
4	The slope of the straight line parallel to the Y – axis is .... A) Positive B) Negative C) Zero D) Undefined	
5	Slope of straight line passes through (3 , 2) and (– 5 , 3) is .... A) $\frac{1}{8}$ B) $-\frac{1}{8}$ C) 8 D) – 8	
6	Slope of straight line passes through (2 , 6) and (7 , 11) is .... A) 1 B) 2 C) 5 D) 6	
7	**In the opposite figure :** The slope of the straight line L is .... (a) zero. (b) undefined. (c) 1 (d) $\frac{1}{2}$  	
8	(3 , 2) satisfies the relation .... A) $Y + X = 5$ B) $Y – X = 5$ C) $3Y – X = 2$ D) $2X + Y = 1$	
9	If : (a , 1) satisfies the relation : $2X + 3y = 7$, then a = ... A) 2 B) – 2 C) 4 D) 3	
10	If : (2 k , 3 k) satisfies the relation : $X + y = 15$, then k = ... A) 5 B) 3 C) - 5 D) -3	

11	The slope of horizontal line is .... A) 1 B) Zero C) - 1 D) Undefined
12	If A (3 , 5) , B (5 , – 1) , then the slope of $\overleftrightarrow{AB}$ = ... A) – 3 B) $-\frac{1}{3}$ C) 3 D) $\frac{1}{3}$
13	If the Slope of straight line a X + b y + 1 = 0 is undefined , then .... A) a = b B) a = zero C) b = zero D) a = – b
14	(3 , 2) does not satisfy the relation .... A) Y + X = 5 B) X – Y = 1 C) Y + X = 7 D) 3 Y – X = 3
15	If : (2 , b) satisfies the relation : 3 X + y = 9 , then b = ... A) 6 B) 3 C) 2 D) 0
16	If : (2 m , m) satisfies the relation : 2 X + 3 y = 35 , then m = ... A) 7 B) 5 C) 14 D) 10
17	If A (3 , 2) , B (0 , 4) , then the slope of $\overleftrightarrow{AB}$ = ... A) - 2 B) 2 C) $\frac{1}{2}$ D) $-\frac{1}{2}$
18	Slope of straight line passes through (3 , 8) and (0 , 2) is .... A) 1 B) 2 C) 1 D) $\frac{1}{2}$
19	Relation : X – 5 = 0 is represented by a st. line whose slope is............ A) 0 B) – 5 C) 5 D) Undefined
20	Value of b where (– 3 , 2) satisfies the relation : 3 X + by = 1 is A) 3 B) 5 C) 4 D) 0
21	If : (a , 4) satisfies the relation : X – y = - 1 , then a = ... A) $\sqrt{3}$ B) 3 C) 27 D) 1.5
22	The opposite table shows the relation between x and y , which is .... (a) y = x + 4 (b) y = x + 1 (c) y = 2 x – 1 (d) y = 3 x – 2

x	1	2	3	4	5
y	1	3	5	7	9

23 Slope of straight line passes through (- 2 , 3) and (2 , 3) is ....

A) 2 B) 1 C) Zero D) Undefined

24 Slope of straight line passes through (2 , 3) and (4 , 7) is ....

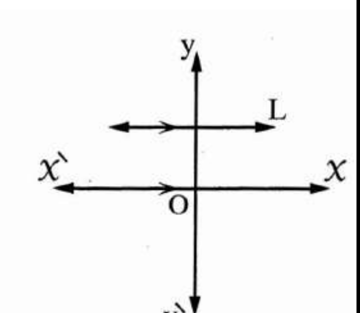
A) 3 B) 2 C) 1 D) $\frac{1}{2}$

25 In the opposite figure :

The slope of the straight line L is ....

(a) positive. (b) negative.

(c) zero. (d) undefined.



26 Which of the following represent linear relation ?

A) $Xy = 2$ B) $X^2 = \frac{1}{y}$ C) $\frac{X}{y} = 1$ D) $y = X^2 + 4$

27 If : (2 , - 5) satisfies the relation : $3X - Y + c = 0$, then c = ...

A) 1 B) – 1 C) 11 D) – 11

28 If : (a , 2 a) satisfies the relation : $y = X - 1$, then a = ...

A) 1 B) 10 C) - 1 D) 3

29 The slope of the straight line parallel to the X – axis is ....

A) Positive B) Negative C) Zero D) Undefined

30 Slope of straight line passes through (- 3 , 1) and (2 , 5) is ....

A) $\frac{4}{5}$ B) $-\frac{6}{1}$ C) $\frac{5}{4}$ D) $-\frac{1}{6}$

31 Slope of straight line passes through (3 , y) and (5 , – 2) is – 3 , then y = ...

A) 2 B) 4 C) 6 D) – 30

32



The slope of the straight line L in the opposite figure is ....

(a) positive. (b) negative.

(c) zero. (d) undefined.

[B] Essay Problems :

1	**Represent graphically the relation :** $y + 2x = 5$ **2016 Exam (4) Question (5) (a)**
2	**Represent the relation :** $y - x = 2$ graphically **2016 Exam (5) Question (5) (a)**
3	**Represent the relation :** $y = x + 2$ graphically. **2016 Exam (3) Question (5) (a)**
4	**Graph :** $2x - y = 3$ does the point (1 , 2) belong to the straight line ? **2016 Exam (10) Question (4) (a)**
5	If (a , 2 a) satisfies $y = x - 1$, **find the value of :** a **2016 Exam (3) Question (3) (b)**
6	**Find the value of :** k where (k , 2 k) satisfies the relation : $x + y = 15$ **2016 Exam (11) Question (4) (a)**
7	If (a , 3) satisfies the relation : $y = 2x - 1$ **find the value of a** **2016 Exam (15) Question (3) (b)**
8	**Using the linear relations , complete the following tables :** (see below) **Exercise (11) Question (4)**
9	📖 Graph the relation : $2x + 3y = 6$ If the straight line representing this relation intersects the x-axis at point A and the y-axis at point B , find the area of the triangle OAB where O is the origin point. « 3 square units » **Exercise (11) Question (13)**

Question 8:

(1) $4x - y = -1$

x	0	1	2	3
y	....	....	....	....

(2) $y = 5x + 15$

x	− 4	− 3	− 2
y	....	....	....

UNIT 3 LESSON (1) Collecting and organizing data

Example (2)

The following frequency table shows the weekly wages of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the descending cumulative frequency table and represent it graphically , then find :

1 The number of workers whose weekly wages are 60 pounds or more.

2 The percentage of the number of workers whose weekly wages are 60 pounds or more.

Solution

- Form the descending cumulative frequency table as follows :

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (Frequency)	5	12	22	7	4

54 and more = 5 + 12 + 22 + 7 + 4 = 50

58 and more = 12 + 22 + 7 + 4 = 45

62 and more = 22 + 7 + 4 = 33

66 and more = 7 + 4 = 11

70 and more = 4

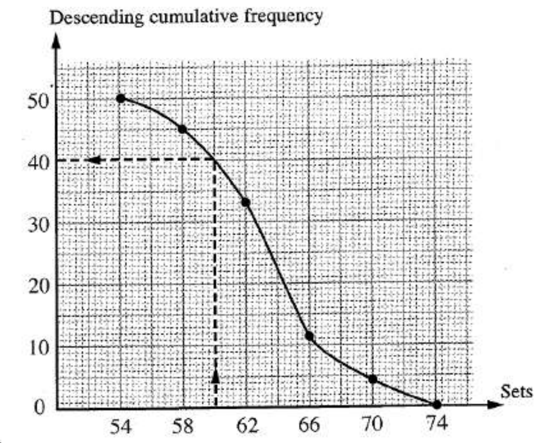
74 and more = 0

The lower boundaries of sets	Frequency
54 and more	50
58 and more	45
62 and more	33
66 and more	11
70 and more	4
74 and more	zero

The descending cumulative frequency table

Notice that : The descending cumulative frequency begins with the total frequency and ends with zero.

- To represent this table graphically , follow the same previous steps in the ascending cumulative frequency table to get the opposite graph.



The descending cumulative frequency curve

- From the graph , we find that :

1 The number of workers whose weekly wages are 60 pounds or more = 40 workers.

2 The percentage of those workers $= \frac{40}{50} \times 100\% = 80\%$

Example 1

The following frequency table shows the weekly wages in pounds of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the ascending cumulative frequency table and represent it graphically , then find :

1. The number of workers whose weekly wages are less than 60 pounds.
2. The percentage of the number of workers whose weekly wages are less than 60 pounds.

Solution

- Form the ascending cumulative frequency table as follows :

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (Frequency)	5	12	22	7	4

Less than 54 = 0
Less than 58 = 5 + 0 = 5
Less than 62 = 5 + 12 = 17
Less than 66 = 5 + 12 + 22 = 39
Less than 70 = 5 + 12 + 22 + 7 = 46
Less than 74 = 5 + 12 + 22 + 7 + 4 = 50

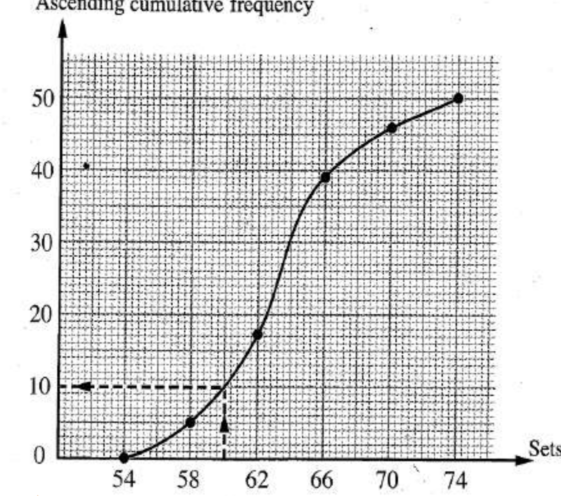
The upper boundaries of sets	Frequency
Less than 54	zero
Less than 58	5
Less than 62	17
Less than 66	39
Less than 70	46
Less than 74	50

The ascending cumulative frequency table.

Notice that : The ascending cumulative frequency begins with zero and ends at the total frequency.

To represent the ascending cumulative frequency table graphically , do as follows :

1. Specialize the horizontal axis for sets and the vertical axis for the ascending cumulative frequency.
2. Choose a suitable scale to represent data on the vertical axis so that it contains the ascending cumulative frequency easily.
3. Represent the ascending cumulative frequency of each set , then draw the graph (the curve) such that it passes through the points which we located as shown in the opposite figure.



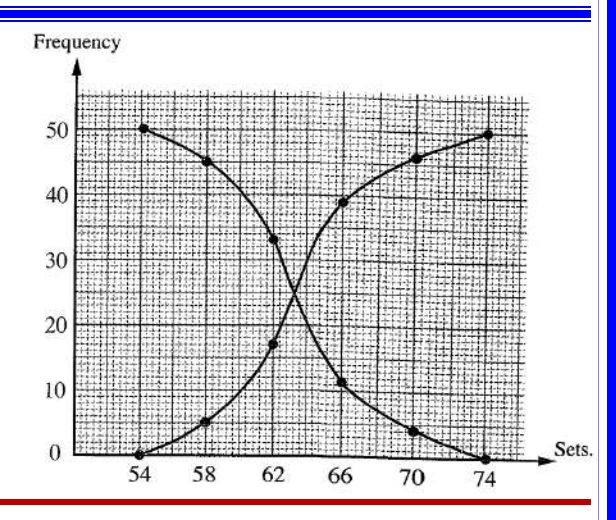
The assending cumulative frequency curve

- From the graph , we find that :

1. The number of workers whose weekly wages are less than 60 pounds = 10 workers.
2. The percentage of the number of workers whose weekly wages are less than 60 pounds $= \frac{10}{50} \times 100\% = 20\%$

Remark

we can graph the two curves of the ascending and descending cumulative frequency of a frequency distribution in one sketch as shown in the opposite graph.



UNIT 3 LESSON (3) Mean

Remember that :

To calculate the mean of a set of values, do as follows :

1. Find the sum of these values.
2. Divide this sum by the number of these values

i.e. The mean of a set of values $= \dfrac{\text{The sum of values}}{\text{Number of values}}$

For example:

If the marks of 5 students are 25 , 23 , 21 , 22 , 24

, then the mean of marks $= \dfrac{25 + 23 + 21 + 22 + 24}{5} = 23$ marks.

Notice that : $23 \times 5 = 115$

, the sum of marks of the 5 students $= 25 + 23 + 21 + 22 + 24 = 115$

i.e. The mean is the value which is given to each item of a set , then the sum of these new values is the same sum of the original values.

Finding the mean of data from the frequency table with sets

Example **The following table shows the distribution of the marks of 50 students in mathematics :**

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

Find the mean of these marks.

1 Determine the centres of sets according to the rule :

$$\text{The centre of a set} = \frac{\text{the lower limit} + \text{the upper limit}}{2}$$

, then the centre of the first set = $\frac{10 + 20}{2} = 15$

, the centre of the second set = $\frac{20 + 30}{2} = 25$ ... and so on.

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

, then its centre = $\frac{50 + 60}{2} = 55$

2 **Form the vertical table :**

Set	Centre of the set « X »	Frequency « f »	$X \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

3 The mean = $\frac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \frac{1700}{50} = 34$ marks.



UNIT 3 LESSON (4) Median





The median is the middle value in a set of values after arranging it ascendingly or descendingly. such that the number of values which are less than it is equal to the number of values which are greater than it.

• To find the median of a set of values , we do as follows :

We arrange the values ascendingly or descendingly

If the values number is odd , then

The median is the value lying in the middle exactly.

For example :

If the values are

42 , 23 , 17 , 30 and 20

We arrange them ascendingly as follows

17 , 20 , (23) , 30 , 42

The median = 23

If the values number is even , then

The median

$= \frac{\text{The sum of the two values lying in the middle}}{2}$

For example :

If the values are

27 , 13 , 23 , 24 , 13 , 21

We arrange them ascendingly as follows

13 , 13 , [21 , 23] , 24 , 27

The median $= \frac{21 + 23}{2} = 22$

Finding the median of a frequency distribution with sets graphically

Example **The following table shows the frequency distribution of marks of 50 students in math exam :**

Sets of marks	0 –	10 –	20 –	30 –	40 –	50 –	Total
Number of students	2	5	8	19	14	2	50

Find the median mark of the student.

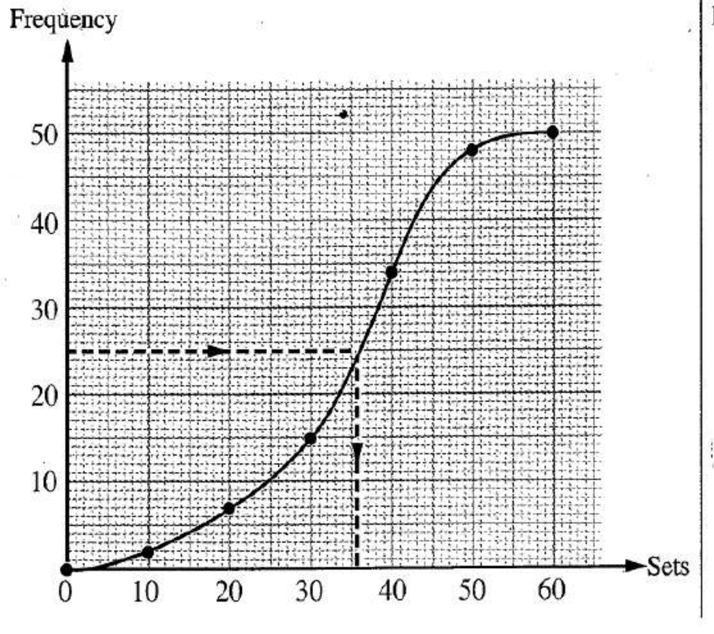
Solution

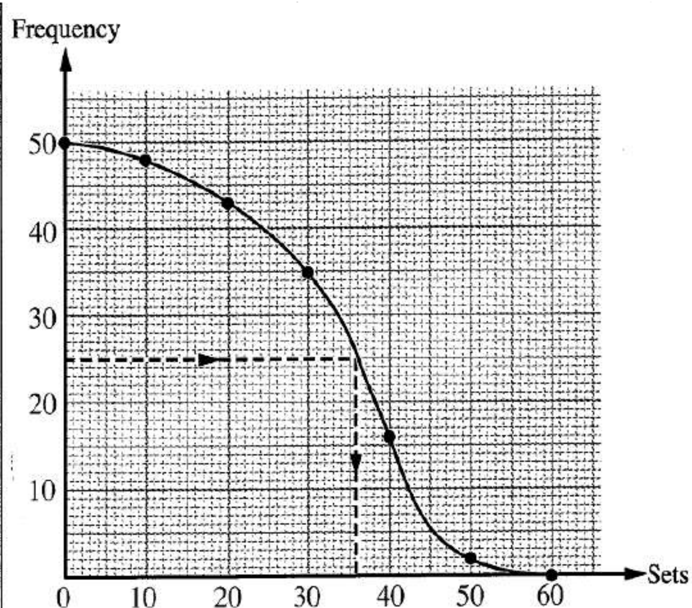
Using the ascending cumulative frequency curve :

The upper boundaries of sets	Frequency
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50

Using the descending cumulative frequency curve :

The lower boundaries of sets	Frequency
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0





$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ From the two previous graphs , the median = 36 approximately

UNIT 3 LESSON (5) Mode



Remember that

The mode of a set of values is the most common value in the set, or in other words, it is the value which is repeated more than any other values.

For example :

The mode of the set of the values : 7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is 7

Finding the mode for a frequency distribution with equal sets in range.

The following is an example which shows how to find the mode of a frequency distribution with sets.

Example

The following is the frequency distribution of marks of 100 pupils in one of the exams :

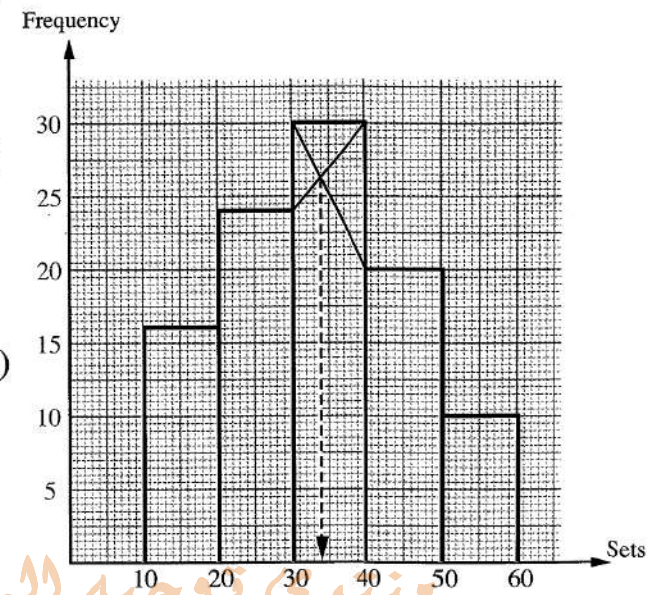
Set of marks	10 –	20 –	30 –	40 –	50 –	**Total**
Number of pupils	16	24	30	20	10	**100**

Find the mode mark for these pupils.

Solution

We can find the mode of that distribution graphically using the histogram as follows :

1. Draw two orthogonal axes : one of them is horizontal and the other is vertical to represent the frequency of each set.
2. Divide the horizontal axis into a number of equal parts with a suitable drawing scale to represent the sets.
3. Divide the vertical axis into a number of equal parts with a suitable drawing scale to represent the greatest frequency in the sets.
4. Draw a rectangle whose base is set (10 –) and its height equals the frequency (16)
5. Draw a second rectangle adjacent to the first one whose base is set (20 –) and its height equals the frequency (24)

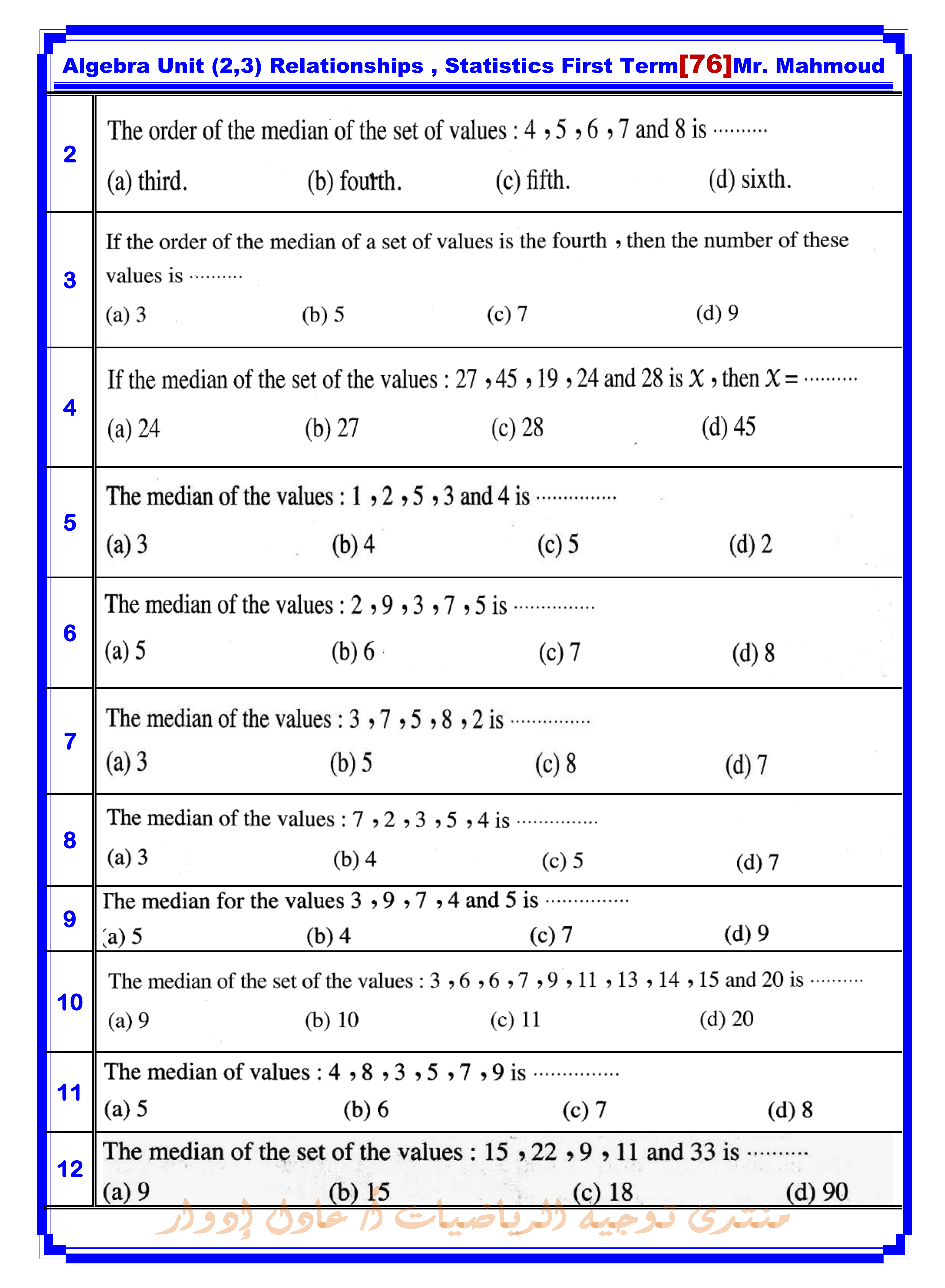


EXERCISES

[A] Choose the correct answer:

1	The arithmetic mean of : 3 , 10 , 2 is .... (a) 10 (b) 5 (c) 3 (d) 6
2	The mean of the values : 2 , 5 , 4 , 5 is .... (a) 4 (b) 5 (c) 16 (d) 8
3	The mean of the values : 2 , 8 , 6 , 4 is .... (a) 2 (b) 5 (c) 4 (d) 6
4	The arithmetic mean of : 3 , 7 , 28 , 52 , 10 = ... (a) 17 (b) 19 (c) 20 (d) 27
5	The arithmetic mean of the values : 19 , 32 , 21 , 6 , 12 is .... (a) 90 (b) 32 (c) 18 (d) 6
6	The mean of the values : 7 , 15 , 19 , 14 and 15 is .... (a) 14 (b) 15 (c) 16 (d) 17
7	The arithmetic mean of the values : 30 , 23 , 25 , 30 , 22 is .... (a) 22 (b) 23 (c) 24 (d) 26
8	If the arithmetic mean of the values : 27 , 8 , 16 , 24 , 6 and k is 14 , then k = ... (a) 3 (b) 6 (c) 27 (d) 84
9	If the mean of marks of 5 pupils is 20 , then the total of their marks = ... marks. (a) 4 (b) 15 (c) 25 (d) 100
10	If the sum of 5 numbers equals 30 , then the arithmetic mean of these numbers is .... (a) 150 (b) 6 (c) 18 (d) 72

11	The set which its lower boundary is 2 and its upper boundary is 6 , then its centre is .... (a) 2 (b) 6 (c) 4 (d) 8
12	The lowest limit of a set is 4 and the other limit is 8 , then its centre is .... (a) 2 (b) 4 (c) 6 (d) 8
13	If the lower limit of a set is 6 and the upper limit is 10 , then its centre is .... (a) 4 (b) 6 (c) 10 (d) 8
14	If the upper limit of a set is 19 and the lower limit of the same set is 11 , then the centre is .... (a) 10 (b) 15 (c) 20 (d) 30
15	If the lowest boundary of a set is 10 and the upper boundary is X and its centre is 15, then X = ... (a) 10 (b) 15 (c) 20 (d) 30
16	If the lower limit of a set is 18 and its centre is 20 , then its length is .... (a) 2 (b) 19 (c) 22 (d) 4
17	The arithmetic mean of the values : 3 – a , 5 , 1 , 4 , 2 + a equals .... (a) 1 (b) 2 (c) 3 (d) 15
18	If the arithmetic mean of the values : 9 , 6 , 5 , 14 , k is 7, then k = ... (a) 1 (b) 5 (c) 34 (d) 35
19	The mean of the values : 2 – a , 4 , 1 , 5 , 3 + a is .... (a) 1 (b) 2 (c) 3 (d) 15
1	The order of the median of the set of values : 8 , 4 , 7 , 6 , 5 is .... (a) 7 (b) 6 (c) 3 (d) 5



2	The order of the median of the set of values : 4 , 5 , 6 , 7 and 8 is .... (a) third. (b) fourth. (c) fifth. (d) sixth.
3	If the order of the median of a set of values is the fourth , then the number of these values is .... (a) 3 (b) 5 (c) 7 (d) 9
4	If the median of the set of the values : 27 , 45 , 19 , 24 and 28 is X , then $X =$ .... (a) 24 (b) 27 (c) 28 (d) 45
5	The median of the values : 1 , 2 , 5 , 3 and 4 is .... (a) 3 (b) 4 (c) 5 (d) 2
6	The median of the values : 2 , 9 , 3 , 7 , 5 is .... (a) 5 (b) 6 (c) 7 (d) 8
7	The median of the values : 3 , 7 , 5 , 8 , 2 is .... (a) 3 (b) 5 (c) 8 (d) 7
8	The median of the values : 7 , 2 , 3 , 5 , 4 is .... (a) 3 (b) 4 (c) 5 (d) 7
9	The median for the values 3 , 9 , 7 , 4 and 5 is .... (a) 5 (b) 4 (c) 7 (d) 9
10	The median of the set of the values : 3 , 6 , 6 , 7 , 9 , 11 , 13 , 14 , 15 and 20 is .... (a) 9 (b) 10 (c) 11 (d) 20
11	The median of values : 4 , 8 , 3 , 5 , 7 , 9 is .... (a) 5 (b) 6 (c) 7 (d) 8
12	The median of the set of the values : 15 , 22 , 9 , 11 and 33 is .... (a) 9 (b) 15 (c) 18 (d) 90

No.	Question
13	The median of the values : 10 , 9 , 11 , 19 , 12 is .... (a) 9 (b) 10 (c) 11 (d) 19
14	The median of the set of the values : 15 , 22 , 9 , 11 and 33 is .... (a) 9 (b) 15 (c) 18 (d) 90
15	The median of the values : 34 , 23 , 25 , 40 , 22 , 14 is .... (a) 22 (b) 33 (c) 24 (d) 25
16	The median of the values : 41 , 23 , 15 , 30 , 20 is .... (a) 23 (b) 15 (c) 30 (d) 20
1	The mode of the values : 3 , 5 , 3 , 6 , 3 and 8 is .... (a) 3 (b) 5 (c) 6 (d) 8
2	The mode of the sets of values : 14 , 11 , 10 , 11 , 14 , 15 , 11 is .... (a) 14 (b) 11 (c) 15 (d) 10
3	If the mode of the set of the values : 4 , 11 , 8 , $2x$ is 4 , then $x =$ .... (a) 2 (b) 4 (c) 6 (d) 8
4	The mode of the values : 15 , 9 , $x + 1$, 9 , 15 is 9 , then $x =$ .... (a) 9 (b) 14 (c) 10 (d) 8
5	The mode of 7 , 8 , 9 , $x + 2$ and 6 is 9 then $x =$ .... (a) 4 (b) 5 (c) 6 (d) 7
6	The mode of : 5 , 6 , 7 , $x + 2$ and 8 is 7 , then $x =$ .... (a) 7 (b) 6 (c) 4 (d) 5
7	If the mode of the set of values : 4 , 11 , $x + 3$, 6 is 6 , then $x =$ .... (a) 2 (b) 3 (c) 4 (d) 6
8	The mode of the set of values : 5 , 9 , 5 , $x - 2$, 9 is 9 , then $x =$ .... (a) 5 (b) 57 (c) 9 (d) 11

[B] Essay Problems :

1

Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	10	12	10	5	40

2016 Exam (1) Question (5) (b)

2

Find the mean of the following data :

Sets	8 –	12 –	16 –	20 –	24 –	Total
Frequency	4	10	16	12	8	50

2016 Exam (2) Question (5) (b)

3

The following table shows frequency distribution of marks of 32 students in an exam :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	3	6	10	8	5	32

Find the mean of this distribution.

2016 Exam (4) Question (5) (b)

4

The following table shows the frequency distribution , find the arithmetic mean :

Sets	10 –	20 –	30 –	40 –	50 –
Frequency	10	20	25	30	15

2016 Exam (8) Question (5) (b)

HOMEWORK

[A] Essay Problems :

1

Find the arithmetic mean of the following frequency distribution :

Sets	1 –	3 –	5 –	7 –	9 –	Total
Frequency	4	6	8	7	5	30

2016 Exam (3) Question (5) (b)

2

Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	10	12	10	5	40

2016 Exam (1) Question (5) (b)

3

Using the following frequency distribution to find the mean :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

2016 Exam (12) Question (5) (b)

4

Using the following distribution , find the arithmetic mean :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

2016 Exam (10) Question (5) (b)

5

Find the mean of the following data :

Sets	8 –	12 –	16 –	20 –	24 –	Total
Frequency	4	10	16	12	8	50

2016 Exam (2) Question (5) (b)

6

The following table shows frequency distribution of marks of 32 students in an exam :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	3	6	10	8	5	32

Find the mean of this distribution.

2016 Exam (4) Question (5) (b)

7

The following table shows the frequency distribution , find the arithmetic mean :

Sets	10 –	20 –	30 –	40 –	50 –
Frequency	10	20	25	30	15

2016 Exam (8) Question (5) (b)

HOMEWORK

[A] Essay Problems :

1

The following table shows the frequency distribution for the scores of 50 students in an examination :

Sets	2–	6–	10–	14–	18–	22–	26–	Total
Frequency	3	5	9	10	12	7	4	50

Find : (1) The mean of the student's score.

(2) The median. « 16.8 , 17.6 »

2

From the following frequency table with equal sets in range :

Sets	10–	20–	x–	40–	50–	60–	Total
Frequency	10	17	20	32	k + 2	4	100

(1) Find the value of each of x and k « $x = 30$, k = 15 »

(2) Graph the ascending and descending cumulative curves on one figure , then calculate the median. « 41 »

3

The following table shows the frequency distribution for the scores of 50 students in an examination :

Sets	2–	6–	10–	14–	18–	22–	26–	Total
Frequency	3	5	9	10	12	7	4	50

Find : (1) The mean of the student's score.

(2) The median. « 16.8 , 17.6 »

4

From the following frequency table with equal sets in range :

Sets	10–	20–	x–	40–	50–	60–	Total
Frequency	10	17	20	32	k + 2	4	100

(1) Find the value of each of x and k « $x = 30$, k = 15 »

(2) Graph the ascending and descending cumulative curves on one figure , then calculate the median. « 41 »

EXERCISES

[A] Complete the Following :

1	The most common value in a set is called ....
2	The value which is the most common of a set of values is called ....
3	The mode of a set of values is ....
4	The mode of the values : 2 , 5 , 1 , 4 , 2 is ....
5	The mode of the values : 4 , 7 , 5 , 7 , 6 , 8 , 7 , 5 is ....
6	The mode of the values : 8 , 7 , 8 , 7 , 6 , 5 , 8 is ....
7	The mode of the set of values : 13 , 12 , 4 , 13 is ....
8	The mode of the set of the values : 14 , 11 , 10 , 11 , 14 , 15 , 11 is ....
9	The mode of the values : 11 , 13 , 11 , 14 , 11 , 12 is ....
10	The mode of the set of the values : 14 , 11 , 15 , 11 , 14 , 15 , 11 is ....
11	The mode value of : 13 , 23 , 46 , 33 , 46 , 43 , 33 , 46 , 32 is .... cm.
12	If the mode of the set of the values : 4 , 5 , a , 3 is 4 , then a = ...
13	If the mode of the values : 3 , 6 , a , 2 , 5 is 6 , than a = ...
14	If the mode of the set of the values : 4 , 5 , a and 3 is 3 , then a = ...

15	If the mode of the values : 5 , 7 and $x + 1$ is 7 , then $x =$ ………
16	The mode of the values : 14 , 8 , $x + 1$, 8 , 14 is 8 , then $x =$ ……………
17	If the mode of the values : 12 , 7 , $x + 1$, 7 , 12 is 7 , then $x =$ ………
18	If the mode of the set of the values : 15 , 9 , $x + 1$, 9 and 15 is 9 , then $x =$ ………
19	If the mode of the set of the values : 15 , 9 , $x + 6$, 9 and 15 is 9 , then $x =$ ………
20	If the mode of the values : 4 , 11 , 8 , and 2^x is 4 , then $x =$ ………

HOMEWORK

[A] Essay Problems :

1 **The following table shows the frequency distribution with equal range sets for the weekly wages of 100 workers in a factory :**

Sets of wages in L.E.	70 –	80 –	90 –	100 –	x –	120 –	130 –
Number of workers	10	13	k – 4	20	16	14	11

Find : (1) The value of each of x and k « $x = 110$, k = 20 »

(2) The mode of wages in L.E. « 105 pounds »

2 **The following is the frequency distribution of the weekly bonus of 100 workers in a factory :**

Bonus in L.E.	20 –	30 –	40 –	50 –	60 –	70 –
No. of workers	10	k	22	26	20	8

(1) Calculate the value of k « 14 »

(2) Find the mean of this distribution. « 50.6 pounds »

(3) Find the mode value of the weekly bonus using the histogram. « 54 pounds »

3

The following table shows the frequency distribution for the weights of 50 students in kg. at a school :

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
Number of students	7	3 k	4 k	10	8	4	50

(1) Find the value of k « 3 »

(2) Calculate the mean. « 44 kg. »

(3) Draw the ascending cumulative frequency curve.

(4) Draw the histogram and find the mode of weights. « 43 kg. »

(5) Find the median. « 43.5 kg. »

4

The following table shows the frequency distribution for the scores of 50 students in an examination :

Sets	2 –	6 –	10 –	14 –	18 –	22 –	26 –	Total
Frequency	3	5	9	10	12	7	4	50

Find : (1) The mean of the student's score.

(2) The median. « 16.8 , 17.6 »

5

From the following frequency table with equal sets in range :

Sets	10 –	20 –	X –	40 –	50 –	60 –	Total
Frequency	10	17	20	32	k + 2	4	100

(1) Find the value of each of X and k « $X = 30$, $k = 15$ »

(2) Graph the ascending and descending cumulative curves on one figure , then calculate the median. « 41 »

